For all questions, answer choice (E) NOTA means that none of the given answers is correct. Diagrams are not necessarily drawn to scale. Good Luck!

I. Points, Lines, Planes, and Angles

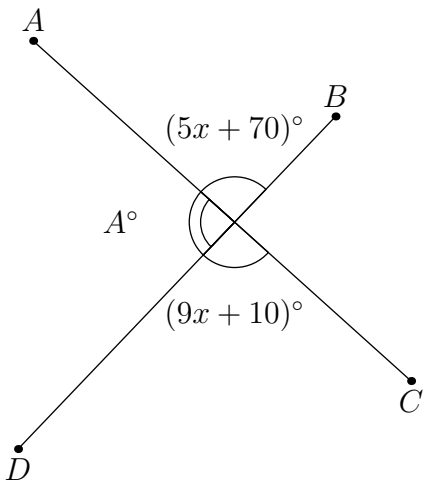
1. Find the sum of the x and y values of the midpoint of $(10, 9)$ and $(8, 7)$.

   (A) 16 (B) 17 (C) 18 (D) 19 (E) NOTA

2. What is the distance between the two points mentioned in the previous problem?

   (A) $\sqrt{2}$ (B) 4 (C) 3 (D) $2\sqrt{2}$ (E) NOTA

3. The two line segments below intersect, forming two pairs of vertical angles. Find the value of angle A.



   (A) 145° (B) 95° (C) 135° (D) 105° (E) NOTA

4. How many of the following statements are true?

   I. Any three points determine a plane.
   II. A line and a point not on the line determine a plane.
   III. If two planes are perpendicular to the same line, then the two planes are parallel to each other.

   (A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

5. Given that the supplement of the supplement of the complement of the complement of an angle is 70°, find the measure of the angle.

   (A) 20° (B) 110° (C) 160° (D) 50° (E) NOTA

II. Triangles

6. A scalene triangle has angles measuring $(x + 40)°$, $(2x + 60)°$, and 80°. Find the value, in degrees, of the smallest interior angle.

   (A) 20 (B) 30 (C) 40 (D) 50 (E) NOTA

7. A triangle has side lengths measuring 8, 27, and x. How many integral values of x can be used such that the triangle is non-degenerate?

   (A) 15 (B) 16 (C) 17 (D) 18 (E) NOTA

8. An isosceles triangle has a base measuring 14 and a leg measuring 8. Find its perimeter.

   (A) 42 (B) 56 (C) 36 (D) 30 (E) NOTA

9. Side lengths AB and AC, in triangle ABC, both measure lengths of 12. Given that the altitude from A to BC has a length of $\sqrt{119}$, find the perimeter of the triangle ABC.

   (A) 34 (B) 26 (C) 28 (D) 32 (E) NOTA

10. Let $\triangle ABC$ be an isosceles triangle with $AB = AC$. Let D and E be the midpoints of $\overline{AB}$ and $\overline{AC}$, respectively. Given that there exists a point F on ray DE outside of $\triangle ABC$ such that $\triangle BFA$ is similar to $\triangle ABC$, compute $\frac{AB}{\overline{BC}}$.

    (A) 4 (B) $\sqrt{3}$ (C) 2 (D) $\sqrt{2}$ (E) NOTA

III. Quadrilaterals

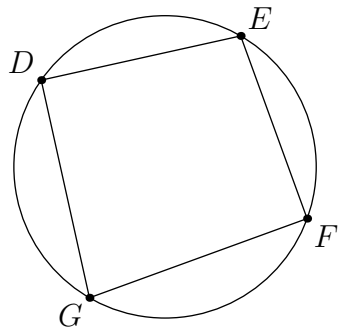
11. The base of a parallelogram has a length of 10 and the height is 19. What is the area of the parallelogram?

    (A) 95 (B) 190 (C) 58 (D) 116 (E) NOTA

12. Find the area of a circle that is inscribed in a square, given that the square's diagonal has a length of 10.

    (A) $\frac{25\pi}{2}$ (B) 25π (C) $\frac{25\pi}{4}$ (D) 50π (E) NOTA

For questions 13 and 14, use the following diagram:



13. Evaluate $\angle DEF + \angle DGF$ in degrees.

    (A) 90 (B) 180 (C) 270 (D) 360 (E) NOTA

14. Given that $DG = 4$, $DE = 6$, $EF = 2$, and $FG = 8$, find the product of the diagonals.

    (A) 42 (B) 64 (C) 56 (D) 72 (E) NOTA

15. A trapezoid has the property that the sum of each pair of opposite sides is 23. The lengths of the legs differ by 7 while the lengths of the bases differ by 17. The area of the trapezoid can be written as a reduced fraction $\frac{x}{y}$ where $x > y$. Evaluate $x - y$.

(A) 1363 (B) 1371 (C) 1337 (D) 1362 (E) NOTA

IV. Other Polygons

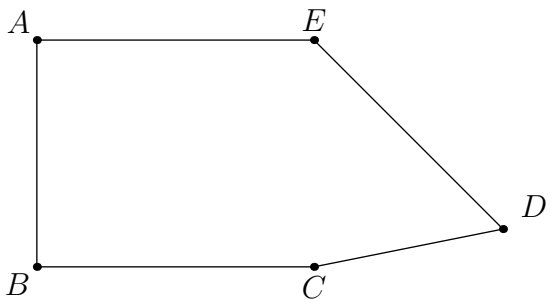
16. Find the sum of all the exterior angles of a regular undecagon, hexagon, and octagon.

(A) 1980° (B) 315° (C) 1080° (D) 360° (E) NOTA

17. A cyclic hexagon has side lengths of 2, 2, 7, 7, 11, and 11, in that order. Find the length of its circumradius.

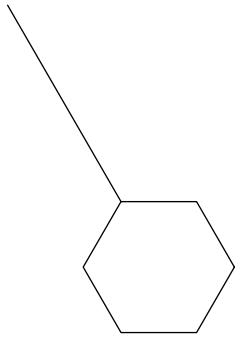
(A) 64π (B) $\frac{169\pi}{4}$ (C) 36π (D) 49π (E) NOTA

18. $AECB$ is a rectangle and EDC is a triangle. The shortest length from AB, which measures 18, to point D measures 37. BC measures 22. The area of $AEDCB$ is x. Find the tens digit of x.



(A) 1 (B) 5 (C) 7 (D) 3 (E) NOTA

19. Larry the Llama is tied to one of the vertices of a regular hexagonal fence. Larry's leash measures 12 and the side length of the fence measures 4. Given that he can't enter the area enclosed by the fence, the total area that Larry has to roam with his leash on is $3A$. Find A. (Assume that he remains on the ground and the length of the rope remains constant.)



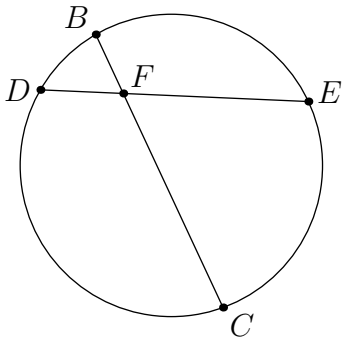
(A) 352π (B) 368π (C) 320π (D) 324π (E) NOTA

20. Using the information from the previous problem, find the area enclosed by the fence.
(A) $24\sqrt{3}$ (B) $144\sqrt{3}$ (C) $216\sqrt{3}$ (D) $36\sqrt{3}$ (E) NOTA

V. Circles

21. Square $ZENG$ has a perimeter of 12 with midpoints Y and I on sides ZE and ZG, respectively. Find the radius of the largest circle that can be inscribed in pentagon $GIYEN$.
(A) $3\sqrt{2}-3$ (B) $2\sqrt{2}$ (C) $\sqrt{3}$ (D) $6-3\sqrt{2}$ (E) NOTA

22. BC and DE are both chords of a circle and intersect at F such that $BF = 6$, $DF = 4$, $FE = 12$. What is the length of BC?
(A) 8 (B) 10 (C) 14 (D) 16 (E) NOTA



23. Triangle LIU is drawn inside a semicircle. The length of LI is equal to the diameter of the semicircle. Point U is on the semicircle. Given that angle L is $39°$, find angle I.
(A) $51°$ (B) $102°$ (C) $71°$ (D) $39°$ (E) NOTA

24. What is the circumference of a circle with radius 5.
(A) 25π (B) 10π (C) 15π (D) 20π (E) NOTA

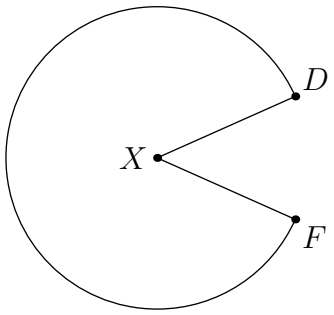
25. There is a semicircle with diameter GI inside square $GINA$. Point X is on GA such that NX is tangent to the semicircle. The area of square $GINA$ is 25 and the length of NX can be expressed as a reduced fraction $\frac{n}{m}$. Find $n+m$.
(A) 9 (B) 11 (C) 13 (D) 16 (E) NOTA

VI. 3D Shapes

26. Pamela has a rectangular prism-shaped box with dimensions $3 \times 7 \times 10$. Find the surface area of the box.
(A) 242 (B) 210 (C) 216 (D) 200 (E) NOTA

27. Find the volume of the Pamela's box, from Question 26.
(A) 105 (B) 200 (C) 201 (D) 110 (E) NOTA

28. Circle X has a radius of 15. Given that angle DXF measures 48°, find the volume of the cone that is formed by aligning the two straight sides.



(A) $\frac{728\pi}{3}$ (B) $\frac{26\pi\sqrt{14}}{3}$ (C) 195π (D) $\frac{338\pi\sqrt{14}}{3}$ (E) NOTA

29. A rectangle with dimensions 6×8 is revolved about its longer side. Find the volume of the resulting figure.

(A) 72π (B) 288π (C) 384π (D) 96π (E) NOTA

30. In tetrahedron $ABCD$, $AB = BC = CA$ and $DA = DB = DC$. Given that the altitude of $ABCD$ from point D is 24 and that the radius of the inscribed sphere of $ABCD$ is 11, determine AB.

(A) 121 (B) 132 (C) 136 (D) 142 (E) NOTA