QUESTION 1

Given the function $f(x) = x^4 - 3x^3 - 28x^2 + 132x - 144$, let

$$\begin{aligned} A &= \text{sum of the } x\text{-intercepts} \\ B &= \text{sum of the roots taken 2 at a time} \\ C &= \text{sum of the roots taken 3 at a time} \\ D &= \text{product of the roots} \end{aligned}$$

Find $B + A(C - D)$.

QUESTION 2

Let

$$
\begin{aligned}
A &= i + 3i^3 + 5i^5 + 7i^7 + \cdots + 15i^{15} \\
B &= i^{163571253} \\
C &= \text{the constant term of } p(x)\text{, if } p(x) \text{ is a fifth-degree polynomial with a leading coefficient of 1, has the roots } 3+2i \\
&\quad \text{and } 1-7i\text{, and the coefficient of } x^4 \text{ is } 12
\end{aligned}
$$

Find $\dfrac{AC}{B}$.

QUESTION 3

Let

$$\begin{aligned} A &= (\log_{64} 128)(\log_4 32) \\ B &= (\log_{27} 81)(\log_9 3) \\ C &= (\log_{10} 100)(\log_{0.01} 1) \\ D &= (\log_6 36)\left(\log_{\frac{1}{4}} \frac{1}{8}\right)\left(\log_{\frac{2}{3}} \frac{8}{27}\right) \end{aligned}$$

Find $A + B + C + D$.

QUESTION 4

For the function $f(x) = \dfrac{x(x^2-25)(x-3)}{(x^2-9)(x+5)}$, the following parts are true or false. Start with 0, and for every true statement add 1, while for every false statement subtract 1. What is the final number?

A = A vertical asymptote is $x = -5$.

B = A horizontal asymptote is $y = 5$.

C = A removable discontinuity is $x = 3$.

D = A slant asymptote is $x - 8$.

QUESTION 5

Let

$$
\begin{aligned}
A &= \sum_{k=0}^{\infty} \frac{2}{3^k} \\
B &= \sum_{k=0}^{\infty} \frac{3^k + 6^k}{9^k} \\
C &= \sum_{k=0}^{20} (k+1)^2 \\
D &= \sum_{k=1}^{100} (-1)^k k^2
\end{aligned}
$$

Find $2AB + C + D$.

QUESTION 6

Let

$$A = \text{the sum of all coordinates of all solutions to the system of equations } x - y = 3,\ \frac{1}{x} + \frac{1}{y} = \frac{1}{2}$$
$$B = \text{the sum of all coordinates of all solutions to the system of equations } x - y = 6,\ 3x + y = 40$$
$$C = \text{the sum of all coordinates of all solutions to the system of equations } 2x + 3y = 35,\ 13x - 6y = -2$$

Find $A(B - C)$.

QUESTION 7

Let

$$
\begin{aligned}
A &= \text{the coefficient of the } x^2y^2 \text{ term in the expansion of } (x-2y)^4 \\
B &= \text{the coefficient of the } x^2 \text{ term in the expansion of } (3x+5)^3 \\
C &= \text{the coefficient of the } x^4y^3 \text{ term in the expansion of } (2x^2-3y)^5 \\
D &= \text{the coefficient of the } x \text{ term in the expansion of } \left(x+\frac{2}{x^2}\right)^4
\end{aligned}
$$

Find $A+B+C+D$.

QUESTION 8

Let $\lceil x \rceil$ be defined as the smallest integer greater than or equal to x. Then let

$$\begin{aligned} A &= \lceil 13.5^2 \rceil \\ B &= \left\lceil \frac{1}{r_1} + \frac{1}{r_2} \right\rceil, \text{ where } r_1 \text{ and } r_2 \text{ are the roots of } x^2 - 11x + 5 \\ C &= \text{the maximum value of } 2^{\lceil -x^2+3x+1 \rceil} \\ D &= \left\lceil \left(\sqrt{3} + \sqrt{2}\right)^4 \right\rceil \end{aligned}$$

Find $A - B - C - D$.

QUESTION 9

Let

$$
\begin{aligned}
A &= \text{the number of positive divisors of 2013} \\
B &= \text{the sum of the first 4 triangular numbers} \\
C &= \text{the units digit of } 2^{2013} \\
D &= xy, \text{ if } x \text{ and } y \text{ are integers such that } 11x + 29y = 1 \text{ and } 0 < x < 37
\end{aligned}
$$

Find $A + B + C + D$.

QUESTION 10

Consider the conic section, $4x^2 - 16x + y^2 + 8y = -16$, for this question. Let

$$\begin{aligned} A &= \text{the area of the conic section} \\ B &= \text{the eccentricity of the conic} \\ C &= \text{the distance between the two foci} \\ D &= \text{the length of the latus rectum} \end{aligned}$$

Find $AD + BC$

QUESTION 11

Consider the functions $f(x) = |3(x+4)|$ and $g(x) = |2 - 2(x-6)|$. They intersect at two points, (A, B) and (C, D).

Find $A + B + C + D$.

QUESTION 12

Let

$$\begin{aligned}A &= \text{the maximum of } 3^{-x^2+2x+3}\\ B &= \text{the maximum of } -8x^2-16x+42\\ C &= \text{the maximum of } 2x^2-6x^2+5x^2-13x+12x-16\\ D &= \text{the maximum of } 2^{-6x^2-12x+6}\end{aligned}$$

Find $A+B+4C+D$

QUESTION 13

Let $A\nabla$ denote the matrix operator $\det\left(A^{-1}\right)$. Then let

$$\begin{aligned} A &= \begin{bmatrix} 4 & 2 \\ 6 & -8 \end{bmatrix}\nabla \\ B &= \begin{bmatrix} -3 & 4 \\ 5 & 2 \end{bmatrix}\nabla \\ C &= \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}\nabla \\ D &= \begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}\nabla \end{aligned}$$

Find $\dfrac{BC}{AD}$.

QUESTION 14

Let

$$\begin{aligned} A &= \text{the number of distinct permutations of RICKARDS} \\ B &= \text{the number of distinct permutations of ALEXYU} \\ C &= \text{the number of distinct permutations of ALGEBRA} \\ D &= \text{the number of distinct permutations of MISSISSIPPI} \end{aligned}$$

Find $A + B + C + D$.