Question 1

Let

$$\begin{aligned} A &= \begin{vmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}, \text{when } \theta = \frac{3\pi}{4} \\ B &= \sum_{n=1}^{179} \cos(n^\circ) \\ C &= \csc^{-1}(\sqrt{2}), \text{in radians.} \\ D &= \tan^2\left(\frac{17\pi}{4}\right) + 1 \end{aligned}$$

Find $A + B + C + D$.

Question 2

For the function, $f(x) = 4\sin(4x + 7) - 8$, find

$$\begin{aligned} A &= \text{the amplitude of the function.} \\ B &= \text{the phase shift of the function.} \\ C &= \text{the period of the function.} \\ D &= \text{the vertical shift of the function.} \end{aligned}$$

Find $ABCD$

Question 3

For the conic whose equation is $(x-7)^2+(y+4)^2=144$, find

$$\begin{aligned} A &= \text{the area bounded by the graph of the conic section.} \\ B &= \text{the shortest distance from the point (-1, -19) to the conic.} \\ C &= \text{the eccentricity of the conic.} \end{aligned}$$

Find $\dfrac{A}{B}+C$

Question 4

Let

$$\begin{aligned} A &= \text{the scalar product (dot product) between the two vectors } 11\vec{i} + 5\vec{j} \text{ and } 7\vec{i} + 15\vec{j} \\ B &= \langle 4, 1, 2\rangle \times \langle 7, -3, -5\rangle \\ C &= \text{the cosine of the angle between the two vectors } \langle 4, 1, 6\rangle \text{ and } \langle 3, 2, 4\rangle \\ D &= \text{the area of the triangle determined by the two vectors } \langle 1, 8, 2\rangle \text{ and } \langle 11, 5, 10\rangle . \end{aligned}$$

Find the unit vector in the direction of $ABCD$, without rationalizing the denominator.

Question 5

For the context of this problem, let $\cos\alpha = \frac{12}{13}, \frac{3\pi}{2} < \alpha < 2\pi$ and $\sin\beta = \frac{24}{25}, \frac{\pi}{2} < \beta < \pi$.

$$\begin{aligned} A &= \cos(\alpha+\beta) \\ B &= \tan(2\alpha) \\ C &= \sin(\beta+\alpha) \\ D &= \text{the area of } \triangle ABC, \text{ when } AB = \frac{119}{12}, AC = 25, \text{ and } \angle A = \beta \end{aligned}$$

Find $\frac{A}{C} + \frac{BD}{323}$

Question 6

Let

$$\begin{aligned} A &= \operatorname{cis}\left(\frac{7\pi}{21}\right)^7 \\ B &= (\sqrt{3}-i)^5 \\ C &= 32e^{\frac{13}{6}\pi i} \\ D &= \text{the sum of the four fourth roots of unity} \end{aligned}$$

Find $A+B+C+D$

Question 7

Where defined, let:

$$\begin{aligned} A &= \sin(x)\cos(x)\sin(2x)\cos(2x)\sec^2(x)\csc^2(x) \\ B &= \frac{1}{2}(\tan^2(x)+1)(\cot^2(x)+1)(\sin 2x)^3 \end{aligned}$$

Evaluate $A^2 + B^2$

Question 8

Let

$$\begin{aligned} A &= \text{the eccentricity of } 4x^2 + 8xy + 4y^2 + 8x - 3y = 45. \\ B &= \text{the radius of } 2x^2 + 2y^2 + 4x + 8y = 40. \\ C &= \text{the length of the minor axis of } 16x^2 + 9y^2 - 128x + 54y + 337 = 144. \\ D &= \text{the length of one latera recta of } 9x^2 - 4y^2 + 36x + 24y = 36 \end{aligned}$$

Evaluate $\dfrac{A+B+C}{D}$

Question 9

Let

$$\begin{aligned} A &= \text{the eccentricity of the polar curve } r = \frac{1}{1+\cos(\theta)} \\ B &= \text{the number of petals in the rose } r = 2012\cos(5\theta) \end{aligned}$$

The polar coordinate, $\left(7, \frac{3\pi}{4}\right)$, when written in rectangular form, is (C, D).

Find $A + B + C + D$

Question 10

Let

$$\begin{aligned} A &= x(t) = 5 + 3t, y(t) = 6t - 8 \\ B &= x(t) = 9 + 4t, y(t) = -8t + 9 \\ C &= x(t) = 2\sin(t) + \frac{37}{4}, y(t) = 2\cos(t) + \frac{21}{2} \end{aligned}$$

Find the shortest distance between the point of intersection of A and B, and the center of C.

Question 11

Find the values of the following series

$$\begin{aligned} A &= \sum_{n=1}^{\infty} \frac{n}{2^n} \\ B &= \sum_{n=1}^{\infty} \frac{n}{3^n} \\ C &= \sum_{n=1}^{\infty} \frac{n}{4^n} \\ D &= \sum_{x=2}^{20} \frac{2}{x^2-1} \end{aligned}$$

Find $\frac{BC}{AD}$

Question 12

Find the sum of the solutions for each of the following equations on the interval $[0, 2\pi)$.

$$\begin{aligned} A &= \tan(\theta)\sin^2(\theta) = 4\tan(\theta) \\ B &= 2\sin(3\theta) - \sqrt{2} = 0 \\ C &= 2\sin^2(\theta) - 3\cos(\theta) = 3 \\ D &= 9 - 9\sin(\theta) = 6\cos^2(\theta) \end{aligned}$$

Find $A + B + C + D$

Question 13

Let

$$\begin{aligned} A &= \lim_{x\to\infty} \frac{7x^3+5x+3}{3x^3+7x^2+x+645} \\ B &= \lim_{x\to\infty} \frac{152x^2+5x+3}{7x^3+2x^2+x+6} \\ C &= \lim_{x\to-\frac{3}{2}} \frac{8x^2+26x+21}{2x^3+x^2-15x-18} \\ D &= \lim_{x\to 0} \frac{\sin(x)}{x} \end{aligned}$$

Find $\frac{AD}{C}+B$

Question 14

Find the number of digits in each of the following numbers, given that $\log 4 \approx .602, \log 7 \approx .845, \log 9 \approx .9542, \log 13 \approx 1.1139$.

$$\begin{aligned} A &= 4^{16} \\ B &= 7^{29} \\ C &= 9^{8} \\ D &= 13^{21} \end{aligned}$$

What is the sum of the digits of $A + B + C + D$?