

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. An inverted cone is filled to capacity and holds A gallons of water. The height of the cone is B . Some water is removed so that C gallons remain. What is the new height of the water level?

(A) $B\sqrt[3]{\frac{C}{A}}$ (B) $\sqrt[3]{\frac{BC}{A}}$ (C) $B\sqrt[3]{\frac{A}{C}}$ (D) $\sqrt[3]{\frac{AB}{C}}$ (E) NOTA

2. Find the smallest natural number with 16 positive factors.

(A) 144 (B) 180 (C) 210 (D) 216 (E) NOTA

3. If $f(x) = Ax^3 + Bx^2 + Cx + D$. $f(0) = 19$, $f(1) = 32$, $f(2) = 207$, and $f(3) = 190$, find $A + B + C$.

(A) 0 (B) 13 (C) 19 (D) 55 (E) NOTA

4. Given positive numbers x and y , which of the following is always true about their arithmetic mean (called A), their geometric mean (called G) and their harmonic mean (called H)?

(A) $A \geq G \geq H$ (B) $A \geq H \geq G$ (C) $G \geq A \geq H$ (D) $G \geq H \geq A$ (E) NOTA

5. Find the maximum value of $\sin \theta \cos \theta \tan \theta$.

(A) -1 (B) 0 (C) 1 (D) No maximum exists (E) NOTA

6. $\int_a^b f(x) dx$ represents the area bounded by the x-axis, the lines $x = a$ and $x = b$, and $f(x)$. Find

$$\int_0^\pi (1 + \cos(x)) dx$$

(A) Impossible to determine (B) 0 (C) π (D) 2π (E) NOTA

7. Find $\sum_{n=1}^{\infty} \frac{1}{a_n}$ where a_n consists of all positive integers whose prime factorization is of the form $2^m 3^n$, where $m, n \geq 0$.

(A) 3 (B) 4 (C) $\frac{10}{3}$ (D) 6 (E) NOTA

8. If Baby Bron-Bron chokes once every 2 games, D. Wade chokes once every 5 games, and a dinosaur chokes once every 9 games, what is the probability that, if the three play together, at least two of them will choke? Consider each event to be independent.

(A) $\frac{29}{45}$ (B) $\frac{13}{90}$ (C) $\frac{7}{45}$ (D) $\frac{1}{6}$ (E) NOTA

9. An Azerbaijani, a Belgian, a Chilean, a Dominican and an Ethiopian are running a race, and all of them place in the top 5. The Dominican beat the Belgian by 2 places. Neither the Chilean nor the Dominican came in 2nd place. The Ethiopian beat the Azerbaijani, but neither came in 3rd. The Chilean finished in the top 3. Between the Azerbaijani and the Belgian, who won, and by how many places?

(A) Azerbaijani, 1 (B) Azerbaijani, 3 (C) Belgian, 1 (D) Belgian, 2 (E) NOTA

10. In AndrewChenLand, weeks have 8 days: Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday, and 8th-day-of-the-week. Today, November 17th, 2012, is a Saturday in AndrewChenLand. On what day of the week will my birthday, May 17th, 2013, fall in AndrewChenLand? Note that a non-leap year in AndrewChenLand still has 365 days.
- (A) 8th-day-of-the-week (B) Monday (C) Tuesday (D) Wednesday (E) NOTA
11. Find the sum of the eigenvalues of
- $$\begin{bmatrix} \pi & 3\sqrt{138} \\ 789 & e \end{bmatrix}$$
- (A) $\pi + e$ (B) πe (C) $\frac{\sqrt{138}}{263}$ (D) $\frac{1 + 2367\sqrt{138}}{789}$ (E) NOTA
12. Which of the following is equivalent to $\cos(76)\cos(75)$?
- (A) $\frac{\cos(1) - \cos(151)}{2}$ (B) $\frac{\cos(1) + \cos(151)}{2}$ (C) $\frac{\cos(151) - \cos(1)}{2}$ (D) $\frac{\sin(1) + \sin(151)}{2}$ (E) NOTA
13. Find the range of the function $f(x) = e^{\log(\sin(x^3+2x^2+4x+8))}$?
- (A) $(-\infty, \infty)$ (B) $(0, e]$ (C) $[1, e]$ (D) $(0, 1]$ (E) NOTA
14. Quadrilateral $ABCD$ has internal angles that increase in an arithmetic progression, with $\angle A$ being the smallest and $\angle D$ being the greatest, and $\angle D = 2\angle A$. Each side of quadrilateral $ABCD$ has length of at least 25 feet. Four cows are tethered to the corners of the quadrilateral with ropes of length 10 feet, one cow per corner. Cows may not roam within the quadrilateral. How much more area can the cow tethered to corner A roam within than the cow tethered to corner B ?
- (A) $\frac{100\pi}{9}$ (B) $\frac{200\pi}{9}$ (C) $\frac{25\pi}{9}$ (D) $\frac{50\pi}{9}$ (E) NOTA
15. Pamela is purchasing papayas, peaches and pineapples. 3 papayas, 2 peaches and 3 pineapples will cost her \$9.00. 2 papayas, 8 peaches and 2 pineapples will cost her \$10.00. 4 papayas, 1 peach and 4 pineapples will cost her \$11.00. How much will 5 papayas, 3 peaches and 5 pineapples cost her?
- (A) \$12.00 (B) \$14.80 (C) \$13.25 (D) \$15.00 (E) NOTA
16. The triangle with vertices $(1, 1)$, $(2, 1)$ and $(2, 2)$ is rotated about the y -axis. How much more volume does this solid contain than the solid achieved by rotating the triangle about the x -axis?
- (A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) NOTA
17. Evaluate $\lim_{x \rightarrow 6^+} \frac{\ln x - \ln 6}{x - 6}$
- (A) $\ln 6$ (B) $\frac{1}{6}$ (C) 0 (D) $\frac{1}{e}$ (E) NOTA
18. Find the next term in this sequence (note that the sequence is not a polynomial): 3, 4, 6, 8, 12, 14, 18, 20, ---
- (A) 22 (B) 23 (C) 24 (D) 26 (E) NOTA

19. Six spheres of radius R sit on a level surface and are externally tangent to one another such that their centers form a regular hexagon. A larger sphere sits on top of the six smaller spheres such that it is externally tangent to each of the six smaller spheres and its lowest point is the center of the aforementioned hexagon. What is the radius of the larger sphere, in terms of R ?

(A) $2R$ (B) $\frac{4R}{3}$ (C) $R\sqrt{3}$ (D) $\frac{3R}{2}$ (E) NOTA

20. A particle is moving along a curve defined by $x = \sin t$, $y = t^{1.5}$ for $t \geq 0$. Find $\frac{dy}{dx}$ at $t = 2\pi$.

(A) undefined (B) $\frac{-8\pi^2\sqrt{2\pi}}{5}$ (C) $\frac{3\sqrt{2\pi}}{2}$ (D) $\frac{3\pi}{4}$ (E) NOTA

21. A unit circle contains two parallel secants that are separated by a distance of $\frac{\sqrt{6} - \sqrt{2}}{2}$. The area between the secants is painted indigo. Find the area of the indigo-painted region.

(A) $\frac{\pi + 3}{6}$ (B) $\frac{\pi + 6}{6}$ (C) $\frac{2\pi + 3}{6}$ (D) $\frac{2\pi + 6}{6}$ (E) NOTA

22. Regular hexagon ABCDEW has side length x . A line is drawn from A to some point on the perimeter of the hexagon that splits its area in a ratio of 2 : 1. Traveling only on the perimeter of the hexagon, how far must one travel to reach this point, in terms of x ? Assume the shortest such path is taken.

(A) $\frac{5x}{2}$ (B) $3x$ (C) $\frac{7x}{2}$ (D) $2x$ (E) NOTA

23. Which of the following is not equivalent to e ?

(A) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ (B) $\sum_{n=0}^{\infty} \frac{1}{n!}$
 (C) The positive value of n that maximizes $n^{\frac{1}{n}}$ (D) $4 \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots\right)$ (E) NOTA

24. Tommy is the troll master! Tommy wants to know the product of 517 and 2483168. What will Tommy's answer be?

(A) 1282797856 (B) 1283797856 (C) 1284797856 (D) 1285797856 (E) NOTA

25. Abraham is the donut master! Abraham's donuts can be modelled by rotating $(x - 3)^2 + y^2 = 4$ about the y-axis. What is the volume of one of Abraham's donuts?

(A) $12\pi^2$ (B) $24\pi^2$ (C) $36\pi^2$ (D) $48\pi^2$ (E) NOTA

26. Alex is the topic test master! The probability of Alex getting a perfect score on a randomly selected topic test is $\frac{4}{5}$. If Alex takes 5 randomly selected topic tests, what is the probability of him getting exactly 3 perfect scores?

(A) $\frac{32}{625}$ (B) $\frac{2}{5}$ (C) $\frac{64}{625}$ (D) $\frac{3}{5}$ (E) NOTA

27. A triangle is drawn. How many of the following points could lie outside of the triangle?

- Centroid
- Circumcenter
- Incenter
- Orthocenter

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

28. An acute angle θ has $\sin \theta = \frac{3}{5}$. Find $\sin \theta + \sin 2\theta + \sin 3\theta$.

(A) $\frac{117}{125}$

(B) $\frac{302}{125}$

(C) $\frac{312}{125}$

(D) $\frac{18}{5}$

(E) NOTA

29. Which of the following vectors is orthogonal to $\langle 6, 1, -2 \rangle$?

(A) $\langle 3, -9, 4 \rangle$

(B) $\langle 0, 0, 1 \rangle$

(C) $\langle 7, -3, 17 \rangle$

(D) $\langle -2, -4, -8 \rangle$

(E) NOTA

30. Evaluate $\int x^5$

(A) $\frac{x^6}{6} + C$

(B) $5x^4 + C$

(C) $\ln(x^{-5}) + C$

(D) $x^6 + C$

(E) NOTA