Question 1

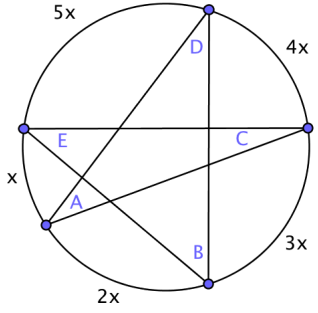
Find the centers of the following circular equations.

$$\begin{aligned} A &= (x+5)^2 + (y-1)^2 = 10 \\ B &= (x-3)^2 + (y-1)^2 = 96 \\ C &= x^2 + 2x + y^2 + 6y = -3 \\ D &= x^2 + 2x + y^2 - 10y = -2 \end{aligned}$$

The centers of A, B, C, and D form a convex polygon. Find the area of that polygon.

Question 2

Given the following diagram, where A, B, C, D, and E are all distinct angles measured in degrees, and the numbers x, 2x, 3x, 4x, and 5x correspond to arc measures of the circle,



Let

$$\begin{aligned} W &= A + B + C + D + E \\ X &= A + C + E \\ Y &= D + B \\ Z &= B \end{aligned}$$

Find $W \times X + Y + Z$

Question 3

Let

A = the sum of the exterior angles minus the sum of the interior angles of a triangle with three distinct angles.

B = the average of the ordinate and the abscissa of the centroid of a triangle with points at (3,-1), (-1,2), and (2,6).

C = the measure of $\angle A$ in degrees if $\triangle ABC$ have area $52\sqrt{2}$ and the side lengths opposite of $\angle B$ and $\angle C$ are 16 and 13, respectively.

D = the area of the intersection of the following three figures: a triangle with vertices at (2,3), (3,4), and (6,-2); a convex quadrilateral with the points (-1,1), (1,2), (2,5), and (-2,2); and a circle with the equation $(x-1)^2 + (y+1)^2 = 4$.

Find $ABCD$

Question 4

Let

$$\begin{aligned} A &= \text{the area of a circle with diameter } \frac{1}{\sqrt{\pi}} \\ B &= \text{the length from a vertex to the incenter of an equilateral triangle with side length 24.} \\ C &= \text{the area of a triangle with an inradius of 5 and a perimeter of 14.} \\ D &= \text{the area of an equilateral triangle with an inradius of } 2\sqrt{3} \end{aligned}$$

Find $(D - 4B) + (C - 4A)$

Question 5

Let

A = the supplement of $\angle C$ of $\triangle ABC$ given that $\angle A + \angle B = 140^\circ$

B = the number of possible values of the third side in a triangle with distinct integer side lengths and two sides equal to 6 and 2 respectively.

C = the length of the space diagonal of a cube, if the diagonal of a face is $4\sqrt{6}$

D = the 56^{th} triangular number.

Find $(D + C) - (A + B)$

Question 6

List out the letters of the true statements as your final answer.

A = Given a fixed surface area, the sphere is the solid that yields the greatest volume.

B = Lines that do not intersect are always parallel.

C = Spherical geometry states that the sum of all angles in a triangle must equal $180°$.

D = One distinct triangle is always defined by knowing two sides and one angle.

E = If a rope is wrapped around the Earth's equator (about 25,000 miles) its length needs to be increased by 2π feet in order to have the rope remain 1 foot away from the equator at every point. Assume that the Earth is perfectly spherical.

Question 7

Let

A = the area of a trapezoid with bases of lengths 2 and 9, and a height of 6.
B = the combined volume of three identical hemispheres with a radius of 3.
C = the ratio of surface area to volume of a cube with side length 4, expressed as a fraction.
D = half of the number of sides on an icosagon.

Find $\frac{D}{A} + BC$

Question 8

Let

A = the area enclosed by the graph of $|4x| + |5y| = 20$.

B = the ratio, expressed as a fraction, of the volume of a cube with side length 6 to the surface area of a cube with side length $12\sqrt{10}$

C = the area of regular hexagon $ABCDEF$, given that point A is at (0,-2) and point D is at (5,3).

Find $AB + C$

Question 9

Triangle ABC has side lengths $AB = 7$, $BC = 6$, and $AC = 11$. Sumkat and Siddarth start simultaneously walking around this triangle in opposite directions from point A at the same speed and they meet at point E.

$$\begin{aligned} A &= \text{the distance traveled by Sumkat.} \\ B &= \text{the distance from point } E \text{ to vertex } B. \\ C &= \text{the area of the triangle.} \\ D &= \text{the inradius of the triangle.} \end{aligned}$$

Find $(A - 2B) + (C - 6D)$

Question 10

Consider a right triangle $\triangle ABC$, with the right angle at B and legs of length 7 and 24. Let

$$\begin{aligned} A &= \text{the length of the median from } B \text{ to } AC \\ B &= \text{the positive difference between the circumradius and the inradius of } \triangle ABC \\ C &= \text{the value of } x, \text{ if } \sqrt{28 \cdot 4 \cdot 21 \cdot x} \text{ is equivalent to the area of } \triangle ABC \\ D &= \text{the value of } \sin(A^\circ + C^\circ) \end{aligned}$$

Find $2(A + B) + CD$

Question 11

There is a recycling bin in the shape of a rectangular prism with the top base removed. It has a length of 40, a width of 45, and a height of 55. John wants to throw a balled up piece of paper in the bin and he always lands the balled up piece of paper within a circle of radius 50.

A = the volume of the bin.
B = the surface area of the bin.
C = the length of the space diagonal of the bin.
D = the probability of John making the shot if the aforementioned circle circumscribes the base of the bin.

Find $D(A - 7B) + C$

Question 12

Let

A = the ratio of the area of circle S to circle R expressed as a fraction, if an arc of 75°on circle S has the same length as an arc of 30°on circle R.

B = the area outside of an equilateral hexagon but inside the circle that circumscribes it given that the hexagon has a side length of 7.

C = Given two objects moving in parallel and opposite directions to each other, with their paths separated by a distance of 15 feet, find the distance in feet between the objects after 5 seconds, given that they start 15 feet apart from each other while one travels at 3 ft/s and the other at 1 ft/s.

Find $AC + B$

Question 13

Let

$$\begin{aligned} A &= \tan 30^\circ \\ B &= \sin 45^\circ \\ C &= \cos 60^\circ \\ D &= x + y + z \text{ in degrees on the interval } [0, 90^\circ], \text{ given that } \sin(x + y + z) = \frac{\sqrt{3}}{2} \end{aligned}$$

Find $A + B + (D)(C^2)$

Question 14

Let

$$\begin{aligned} A &= \text{the area of a circle with radius } 4 \\ B &= \text{the area of a circle with radius } 5 \\ C &= \text{the area of a circle with radius } 2 \\ D &= \text{the area of a circle with radius } 7 \end{aligned}$$

Find $\dfrac{A+B+C+D}{4}$.