For all questions, answer choice (E) NOTA means that none of the given answers for a question are correct. All inverse trigonometric functions have their traditional restricted ranges. Good luck!

1. A particle moves according to the parametric equations $x(t) = t^3 + 3t + 7$ and $y(t) = t^2 + 5t + 3$. Find the speed of the particle at $t = 1$. Note that the formula for speed is $\sqrt{(x'(t))^2 + (y'(t))^2}$.

   (A) $\sqrt{85}$ (B) $\sqrt{69}$ (C) $\sqrt{41}$ (D) 13 (E) NOTA

2. Find $\frac{d}{dx} xe^{5x}$.

   (A) $5xe^{5x}$ (B) $e^{5x} + xe^{5x}$ (C) $5e^{5x} + 5xe^{5x}$ (D) $e^{5x} + 5xe^{5x}$ (E) NOTA

3. On what domain is the graph of $y = 2x^3 - 9x^2 + 12x - 17$ concave down?

   (A) $(1, 3)$ (B) $(-\infty, \infty)$ (C) $\left(-\infty, \frac{3}{2}\right)$ (D) $\left(\frac{3}{2}, \infty\right)$ (E) NOTA

4. At what value of x is the quantity $x^{\frac{1}{\sqrt{x}}}$ maximized, if x is a positive real number?

   (A) e^2 (B) $\sqrt{e}$ (C) $\frac{1}{\sqrt{e}}$ (D) $\frac{1}{e^2}$ (E) NOTA

5. Let $A = \int_1^2 (x^3 + 2)\, dx$, let $B = \int_2^3 (x^3 + 2)\, dx$, let $C = \int_3^4 (x^3 + 2)\, dx$, and let $D = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$. Compute the value of $\frac{A + B + C}{D^2}$.

   (A) $\frac{281}{4}$ (B) $\frac{586}{5}$ (C) 45 (D) $\frac{279}{4}$ (E) NOTA

6. Compute $\lim_{n\to\infty} \ln\left(1 + \frac{e}{n}\right)^{en}$.

   (A) e^e (B) e^2 (C) e^{e^e} (D) e^3 (E) NOTA

7. Evaluate:

$$\int_0^1 e^{4x}\, dx$$

   (A) $\frac{e^4 + e - 1}{4}$ (B) $e^4 + e$ (C) $\frac{e^4 - 1}{4}$ (D) $4e - 1$ (E) NOTA

8. Let $f(x)$ be a function that is continuous and differentiable on the domain of all real numbers. If $f(2) = 8, f(10) = 48$, compute the average value of $f'(x)$ on the interval $[2, 10]$.

   (A) 4 (B) 5 (C) 20 (D) 28 (E) NOTA

9. Find the slope of the tangent line to the curve $3xy - y^2 + 1 = x^2 + x$ at the point $(0, -1)$.

   (A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) 1 (D) 2 (E) NOTA

10. Let $f(x) = \frac{\cos x\left((\sin x + \cos x)^2 - 1\right)}{\cos 2x}$. Compute $f'\left(\frac{\pi}{6}\right)$.

   (A) $-\frac{9}{2}$ (B) $\frac{5\sqrt{3} + 2}{2}$ (C) $\frac{5}{4}$ (D) $\frac{7\sqrt{3}}{2}$ (E) NOTA

11. There is a unique q such that $\int_q^{\infty} \frac{1}{4+x^2}\,dx = \frac{\pi}{3}$. The quantity q^2 can be expressed as $\frac{a}{b}$, where a, b are positive, relatively prime integers. Compute $a+b$.

(A) 13 (B) 9 (C) 4 (D) 7 (E) NOTA

12. Find $\frac{d}{dx}\left[x^2 + 3x \cdot \ln(x+3) - 2^x\right]$ at $x = 1$.

(A) $\frac{11\ln 2 + 16}{8}$ (B) $\frac{11\ln 2 - 14}{4}$ (C) $\frac{16\ln 2 + 11}{4}$ (D) $5 - \ln 2$ (E) NOTA

Questions 13 and 14 deal with the following information:

Let $g(z)$ yield the area of the triangle such that all three of its vertices lie on the graph $y = x^2$, and one of them is at the point $(0,0)$, while the other two are the points (z, z^2) and $(-z, z^2)$.

13. Compute:

$$\int_2^{3\sqrt{2}} g(z)\,dz$$

(A) 154 (B) 146 (C) 77 (D) 73 (E) NOTA

14. Let $h(z) = g^{-1}(z)$. What is the value of a such that $h'(a) = 1$?

(A) $\frac{\sqrt{5}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{5}}{8}$ (D) $\frac{\sqrt{3}}{9}$ (E) NOTA

Questions 15 and 16 deal with the following information:
Define a region S to be bounded by $y = \sqrt{x}$, $y = \sqrt[3]{x}$, and $x = 1$.

15. Find the exact area of S.

(A) $\frac{1}{9}$ (B) $\frac{1}{18}$ (C) $\frac{2}{15}$ (D) $\frac{1}{12}$ (E) NOTA

16. Find the volume of revolution when S is revolved about the x-axis.

(A) $\frac{\pi}{5}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{8}$ (D) $\frac{\pi}{9}$ (E) NOTA

17. Let $a_n = \sum_{k=1}^{n} k^3$, and let $b_n = \sum_{k=1}^{n} nk^2$. Evaluate the following limit:

$$\lim_{n\to\infty} \frac{a_n}{b_n}$$

(A) e^2 (B) $e^{\frac{3}{4}}$ (C) 2 (D) $\frac{3}{4}$ (E) NOTA

18. Find the area of the graph of the polar curve $r = \sqrt{1 + \cos^2\theta}$.

(A) $\frac{3\pi}{2}$ (B) $\frac{8\pi}{3}$ (C) $\frac{15\pi}{8}$ (D) π (E) NOTA

19. If $f(x) = 1 + x + x^2$, find $f'(1) + f'(2) + \cdots + f'(10)$.

(A) 120 (B) 115 (C) 135 (D) 150 (E) NOTA

20. Evaluate:

$$\int_1^{\sqrt{3}} 12x \tan^{-1} x \, dx$$

(A) $6 - 6\sqrt{3} + 5\pi$ (B) $9 + 2\sqrt{3} - 2\pi$ (C) $3\pi\sqrt{3} + 2\sqrt{3} - 4$ (D) $10\pi - 7\sqrt{3} + 4$ (E) NOTA

21. Let $R(x) = H(5x)S(2x)$. If $H(x) = 2x$, and $S(x) = x^2$, compute $R'(1)$.

(A) 40 (B) 60 (C) 80 (D) 120 (E) NOTA

22. The velocity of a particle that travels along the x-axis at time t is given by $v(t) = 3t^2 - 5t$. When $t = 7$, the particle is at $x = 2$. Find its position when $t = 3$.

(A) -214 (B) 12 (C) $\frac{9}{2}$ (D) -120 (E) NOTA

23. Chico has a cylindrical birthday cake. Unfortunately, it's mutated, and its radius is growing at a rate of 1 units per second, and its height is shrinking at a rate of 2 units per second. When its radius and its height are equal to 9, how fast is its volume changing, in cubic units per second?

(A) 0 (B) 32π (C) 81π (D) 100π (E) NOTA

24. Use the Trapezoid Rule on $[1, 3]$ with four equal subintervals to approximate the value of

$$\int_1^3 x^2 \, dx$$

(A) $\frac{35}{2}$ (B) $\frac{39}{2}$ (C) $\frac{35}{4}$ (D) $\frac{39}{4}$ (E) NOTA

25. Let $f(x) = x^3 + x + 1$. Compute $f'(3) - f'(2)$.

(A) 15 (B) 3 (C) 2 (D) 17 (E) NOTA

26. Let $y = f(x)$ be the magnitude of the vector $\langle 1, 5, 3x \rangle$. Compute $\frac{dx}{dy}$ when $x = -3$.

(A) $\frac{\sqrt{107}}{27}$ (B) $\frac{\sqrt{109}}{29}$ (C) $-\frac{\sqrt{111}}{31}$ (D) $-\frac{\sqrt{113}}{33}$ (E) NOTA

27. A point is moving along the graph of $f(x) = x^2 - 1$, where $-1 \leq x \leq 1$. At time t (where $0 \leq t \leq 2$), the point is at point $(t - 1, f(t - 1))$. At time t, let $f_t(x)$ be the function such that $f_t(x) = 2\left(x - (t - 1)\right)^2 + f(t - 1)$, and let $g(t)$ denote the area bounded by $f_t(x)$ and the x-axis. Evaluate $g'\left(\frac{1}{2}\right)$.

(A) $\frac{\sqrt{2}}{2}$ (B) $-\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{6}}{2}$ (D) $-\frac{\sqrt{6}}{2}$ (E) NOTA

28. First, a few facts:

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$s(n) = \int_0^{\infty} \left(\sum_{y=1}^{n} e^{-y^4 x^2} \right) dx$$

Evaluate

$$\lim_{n\to\infty} s(n)$$

(A) $\frac{\pi^{\frac{3}{2}}}{6}$ (B) $\frac{\pi^{\frac{5}{2}}}{12}$ (C) $\frac{24}{\pi}$ (D) $\frac{3\sqrt{\pi}}{2}$ (E) NOTA

29. Define the floor function, $f(x) = \lfloor x \rfloor$, to yield the greatest integer less than or equal to x. If the positive solution to the equation

$$\int_0^A \lfloor Ax \rfloor\, dx = \frac{8}{A}$$

can be expressed as $\sqrt{\frac{a}{b}}$ where a, b are positive, relatively prime integers, compute ab.

(A) 18 (B) 10 (C) 36 (D) 42 (E) NOTA

30. A complex number $e_0 = a + bi$ is uniformly chosen in the Argand plane such that $|a| < 1$ and $|b| < 2$. Let γ be the expected value of the magnitude of e_0^2. Let $\theta°$ be the expected value of the principal value of the argument of e_0. Compute $\gamma\theta$. Note: here the principal value of the argument of e_0 is defined as the argument of e_0 in the interval $[0, 360°)$

(A) 300 (B) 360 (C) 240 (D) 420 (E) NOTA