

For all questions, choice (E) NOTA means that none of the given answers is correct. $i = \sqrt{-1}$.

- Convert $\frac{7\pi}{30}$ to degrees.
(A) 42° (B) 21° (C) 70° (D) 1040° (E) NOTA
- Find the value of $\sin 75^\circ + \cos 75^\circ$.
(A) 1 (B) $\frac{\sqrt{6}}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}$ (E) NOTA
- Triangle $\triangle ABC$ has side lengths of $\overline{BC} = 4$ and $\overline{BA} = 5$. If $\angle B = 60^\circ$, find the area of the triangle.
(A) 5 (B) $10\sqrt{3}$ (C) 10 (D) $5\sqrt{3}$ (E) NOTA
- A point has coordinates $(-3\sqrt{2}, 3\sqrt{2})$ on the Cartesian plane. What are the coordinates of this point on the polar coordinate system?
(A) $(6, \frac{3\pi}{4})$ (B) $(3\sqrt{2}, \frac{3\pi}{4})$ (C) $(-6, \frac{3\pi}{4})$ (D) $(-3\sqrt{2}, \frac{3\pi}{4})$ (E) NOTA
- Which of the following equals $\sin^2(x) - \sin^4(x)$ for all x ?
(A) $\frac{\sin^2(2x)}{4}$ (B) $\frac{\sin^2(2x)}{4}$ (C) $\frac{\cos^2(2x)}{2}$ (D) $\frac{\cos^2(2x)}{4}$ (E) NOTA
- The first four positive natural numbers are elements of a 2×2 matrix. If each element has a distinct value, find the second greatest possible determinant of this matrix.
(A) 12 (B) 10 (C) 8 (D) 6 (E) NOTA
- If $\cos(\theta) = \frac{1}{4}$, what is the product of all possible values of $\sin(\theta)$?
(A) $\frac{-3}{4}$ (B) $\frac{-15}{16}$ (C) $\frac{-1}{4}$ (D) $\frac{-9}{16}$ (E) NOTA
- Compute the dot product of the vectors $\langle 2, 3, -1 \rangle$ and $\langle -1, 0, 4 \rangle$.
(A) -6 (B) -2 (C) 7 (D) 0 (E) NOTA
- What is the focus of the parabola, $y = \frac{1}{4}(x - 2)^2 - 3$?
(A) $(2, -4)$ (B) $(2, -2)$ (C) $(-2, -3)$ (D) $(3, -3)$ (E) NOTA
- For what values of x does $\arctan(\tan(x)) = x$?
(A) $(-\pi, \pi)$ (B) $(\frac{-\pi}{2}, \frac{\pi}{2})$ (C) $[\frac{-\pi}{2}, \frac{\pi}{2}]$ (D) $[-\pi, \pi]$ (E) NOTA
- Evaluate $(\log_6 3)^2 + \frac{\log_6 18}{\log_2 6}$.
(A) $\log 6$ (B) $\log_2 3$ (C) $1 + \log_2 6$ (D) $\log_3 2$ (E) NOTA

12. What is the graph of $x^2 - 3xy + 7y^2 - 2x - 5y = 2$?
- (A) Circle (B) Non-Circular Ellipse
(C) Hyperbola (D) Parabola (E) NOTA
13. Find the area of the ellipse in the Cartesian plane given by the parametric equations $y = 5 \sin(t)$ and $x^2 = 3 \cos^2(t)$.
- (A) $5\pi\sqrt{3}$ (B) $\pi\sqrt{15}$ (C) $3\pi\sqrt{5}$ (D) 15π (E) NOTA
14. If $\log_{10}(\sin \theta) = a$ and $\log_{10}(\cos \theta) = b$, find $\log_{10}((\csc \theta)(\cot \theta))$ in terms of a and b .
- (A) $2 - a - b$ (B) $a - 2b$ (C) $2b - a$ (D) $2a - b$ (E) NOTA
15. Let obtuse triangle ABC have lengths $AB = 10$ and $AC = 12$, such that angle A is obtuse. If $\sin(A) = \frac{\sqrt{3}}{2}$ what is the length of BC ?
- (A) $2\sqrt{91}$ (B) $2\sqrt{31}$ (C) $2\sqrt{19}$ (D) 22 (E) NOTA
16. A quadratic $f(x) = ax^2 + bx + c$ with integral coefficients passes through $(1 + \sqrt{7}, 3)$, and has zeroes r_1 and r_2 . Given that c equals the product of the roots of the quadratic, compute $(r_1)^2 + r_2r_1 + (r_2)^2$.
- (A) -4 (B) 13 (C) 7 (D) 15 (E) NOTA
17. Suppose $\tan(x) = \frac{a}{b}$. If a and b are positive numbers and $0^\circ < x < 90^\circ$, find $\sin(x)$ in terms of a and b .
- (A) $\frac{b}{\sqrt{a^2 + b^2}}$ (B) $\frac{1}{\sqrt{a^2 + b^2}}$ (C) $\frac{1}{\sqrt{1 + (\frac{a}{b})^2}}$ (D) $\frac{1}{\sqrt{1 + (\frac{b}{a})^2}}$ (E) NOTA
18. Let $x = \sqrt{1 + \sqrt{2 + \sqrt{3 + \sqrt{4 + \dots}}}}$, and $y = \sqrt{3 + \sqrt{4 + \dots}}$. Find y in terms of x , given that both exist.
- (A) $y = x^2 - 2x - 1$ (B) $y = x^2 - 2x + 1$ (C) $y = x^4 - 2x^2 - 1$ (D) $y = x^4 - 2x^2 + 1$ (E) NOTA
19. If $f(x) = 2011 \sin(2011x + 2011) + 2011$, find the product of the phase shift, vertical shift, period, and amplitude of $f(x)$.
- (A) 4022π (B) -4022π (C) 2011π (D) -2011π (E) NOTA
20. Given that r is the smaller root of $6x^2 + x - 2 = 0$, find $\sum_{k=0}^{\infty} (5r(3r^2)^k)$.
- (A) -3 (B) 7 (C) -7 (D) 10 (E) NOTA
21. Find the sum of the coefficients of the expansion of $(x + iy)^7$.
- (A) $8 + 8i$ (B) $16 + 16i$ (C) $8\sqrt{2}$ (D) $8 - 8i$ (E) NOTA

22. Find the principal value of x if $\arctan(x) = \arctan\left(\frac{\pi}{7}\right) + \arctan\left(\frac{2\pi}{7}\right)$.
- (A) $\frac{3\pi}{49 - 2\pi^2}$ (B) $\frac{21\pi}{49 + 2\pi^2}$ (C) $\frac{21\pi}{49 - 2\pi^2}$ (D) $\frac{21\pi}{1 - 2\pi^2}$ (E) NOTA
23. Express $r^2 = 2\sin(2\theta)$ in Cartesian form.
- (A) $x^4 + 2xy(xy - 2) + y^4 = 0$ (B) $x^4 - y^4 = 2xy$
 (C) $x^2 - 2xy + y^2 = 0$ (D) $x^4 + 2xy(2 + xy) + y^4 = 0$ (E) NOTA
24. Consider the circle $x^2 + y^2 = 2x + 24$. Let the line tangent to the circle at $(5, 3)$ have slope m_1 and the perpendicular line at $(5, 3)$ have slope m_2 . Find $30 \cdot m_1 \cdot m_2$.
- (A) $\sqrt{5}$ (B) $\sqrt{15}$ (C) $30\sqrt{15}$ (D) -30 (E) NOTA
25. When graphed in the Argand plane, how many solutions of $x^{15} = 15$ are in the second quadrant?
- (A) 3 (B) 4 (C) 5 (D) 6 (E) NOTA
26. Find the domain of $g(x) = \sqrt{\ln(\tan(x))}$, given that x is restricted to $[0, 2\pi)$.
- (A) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$ (B) $(0, 2\pi)$ (C) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right] \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$ (D) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right] \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$ (E) NOTA
27. Find the sum of the infinite sequence $\frac{2+3}{2 \cdot 3} + \frac{4+9}{4 \cdot 9} + \frac{8+27}{8 \cdot 27} + \frac{16+81}{16 \cdot 81} + \dots$
- (A) $\frac{9}{4}$ (B) $\frac{4}{3}$ (C) 2 (D) $\frac{3}{2}$ (E) NOTA
28. Evaluate $\cos \frac{\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{5\pi}{7}$.
- (A) $\frac{3}{2}$ (B) $\sin \frac{6\pi}{7}$ (C) $\frac{1}{2}$ (D) $\cos \frac{11\pi}{7}$ (E) NOTA
29. Let $a_1, a_2, a_3, a_4,$ and a_5 be the five roots of the equation $x^5 - x^3 + 1 = 0$.
- Let
- $$A = (a_1 - 2)(a_2 - 2)(a_3 - 2)(a_4 - 2)(a_5 - 2)$$
- If $B = a_1 a_2 a_3 a_4 a_5$, find $\frac{A}{2B}$.
- (A) -50 (B) 25 (C) 30 (D) 75 (E) NOTA
30. Define the hyper-factorial, which is given by $j = \prod_{i=1}^n (i!)$. Compute the smallest the positive integer n such that j has at least 120 successive zeroes at the end of its decimal expansion.
- (A) 32 (B) 31 (C) 34 (D) 35 (E) NOTA