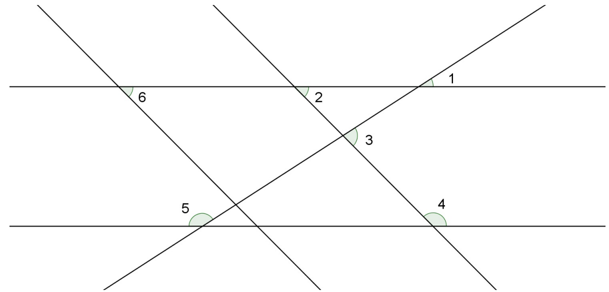
QUESTION 1



In the diagram above, $\angle 1$ and $\angle 5$ are supplementary and $\angle 2 = \angle 6$. If $\angle 1 = 34^\circ$ and $\angle 2 = 55^\circ$, find $\angle 3 + \angle 4 + \angle 5 + \angle 6$.

QUESTION 2

A = The sum of the degrees of the interior angles of a regular pentagon

B = The measure of an exterior angle of a regular 36-gon

C = The measure of the smallest interior angle of a quadrilateral with angles $(3x)^\circ, 60^\circ, (2x+12)^\circ$, and $(200-x)^\circ$.

D = The measure of an interior angle of a regular dodecagon (12 sided polygon) (in degrees)

Find $\frac{A}{B} - C + D$

QUESTION 3

$$\begin{aligned} A &= \text{The area of a regular hexagon with side length 2} \\ B &= \text{The area of an isosceles trapezoid with median 9 and height 4} \\ C &= \text{The area of a rhombus with diagonals 2 and 7} \\ D &= \text{The area of a circle with radius } \frac{2}{\sqrt{\pi}} \end{aligned}$$

Find $A + B - C - D$.

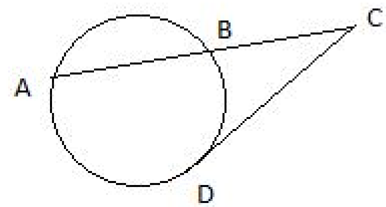
QUESTION 4

The value of each statement is given in parentheses on the left. Find the sum of the values of all true statements.

(2) : The contrapositive of the statement 'If I dance then I sing' is 'If I do not dance then I do not sing'.
(−3) : Skew lines never intersect.
(5) : The total surface area of a cone is equal to $\pi r l$ where r is the radius and l is the slant height of the cone.
(−2) : A regular hexagon has six lines of symmetry.
(1) : If the diagonals of a parallelogram bisect, then it is also a rhombus.
(4) : The decahedron is one of the five platonic solids

QUESTION 5

Use the following diagram to answer parts A and B



$A =$ The measure of the length of BC if $AB = 5$ and $CD = 6$.
$B =$ The measure in degrees of arcBD if arc$AB = 80°$ and $\angle C = 20°$.

An isosceles trapezoid is drawn with base lengths 6 and 10 and height 4. The trapezoid is cut at its median and forms two distinct regions. Let

$C =$ The length of the slant height of the original trapezoid before it is cut.
$D =$ The ratio of the areas of smaller region to the larger region of the cut trapezoid.

Find: $\dfrac{BD}{AC}$

QUESTION 6

A = The greatest possible distance between two vertices (corners) of a cube with side length 6.

B = The perimeter of a rhombus with diagonals of lengths 10 and 24.

C = The least possible value of the perimeter of a right triangle with two sides of lengths 4 and 8.

D = The length of AC of regular hexagon $ABCDEF$ with side length 6.

Find $(A + B) - (C + D)$

QUESTION 7

Annie has a circular dartboard with radius 6. A regular hexagon is inscribed in the circle and then a smaller circle is inscribed within this hexagon.

$A =$ The area of the inscribed hexagon.

$B =$ The ratio of the area of the inner circle to the area of the inscribed hexagon.

$C =$ The area of one of the six regions that is bounded by the hexagon and the outer circle.

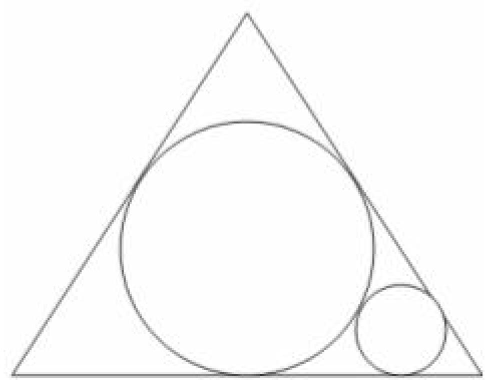
Find $AB + C$

QUESTION 8

An equilateral triangle $\triangle XYZ$ has an inscribed circle with radius 1.

$A =$ The ratio of the area of the inscribed circle to the area of $\triangle XYZ$.
$B =$ The perimeter of the triangle formed by connecting the midpoints of sides XY, YZ, and XZ.

Andrew draws a smaller circle that is tangent to the original inscribed circle and two sides of the triangle as shown below.



$C =$ The length of the radius of the smaller circle.
$D =$ The ratio of the area of the smaller circle to the original inscribed circle.

Find $\dfrac{ABCD}{\pi}$

QUESTION 9

Steve draws a triangle with vertices $(1,-1)$, $(3,3)$, and $(6,-3)$ on a coordinate plane. Meanwhile, Jeremy draws a pentagon with vertices $(2,1)$, $(4,1)$, $(6,3)$, $(4,5)$ and $(1,4)$ on the same coordinate plane.

A = The area of the intersection of the triangle and the pentagon.

B = The area of the union of the triangle and the pentagon.

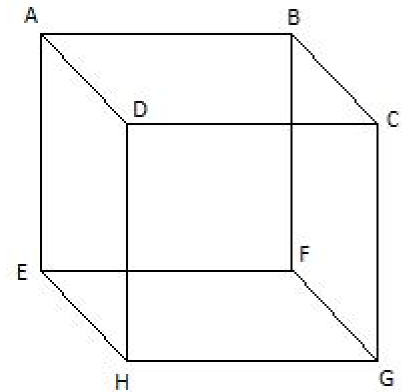
C = The slope of the median drawn from point $(6,-3)$ on Steve's triangle.

D = The sum of the abscissas of the new pentagon formed if Jeremy flips the original pentagon about the y-axis.

Find $A+B+C+D$.

QUESTION 10

The following cube has a side length of 6. X is the midpoint of DH, and Y is the midpoint of CG.



If the cube is sliced by a plane from AB to XY, then let

$A =$ The sum of the total surface areas of the two resulting parts.
$B =$ The positive difference of the volumes of the two resulting parts.
$C =$ The length of diagonal BX.

Find $A + B + C$

QUESTION 11

Pappus' Theorem states that the volume of the solid created by revolving a 2-D shape about a given line is $V = A(D_C)$ where V is the volume of revolution, A is the area of the 2-D shape, and D_C is the distance that the centroid, or center, travels during the revolution.

If the circle $x^2 + y^2 = 16$ is revolved 360° around the line $y = -6$. Then, let

$A =$ The shortest distance between the circle and the line.
$B =$ The volume of the torus (doughnut) formed by the revolved circle.

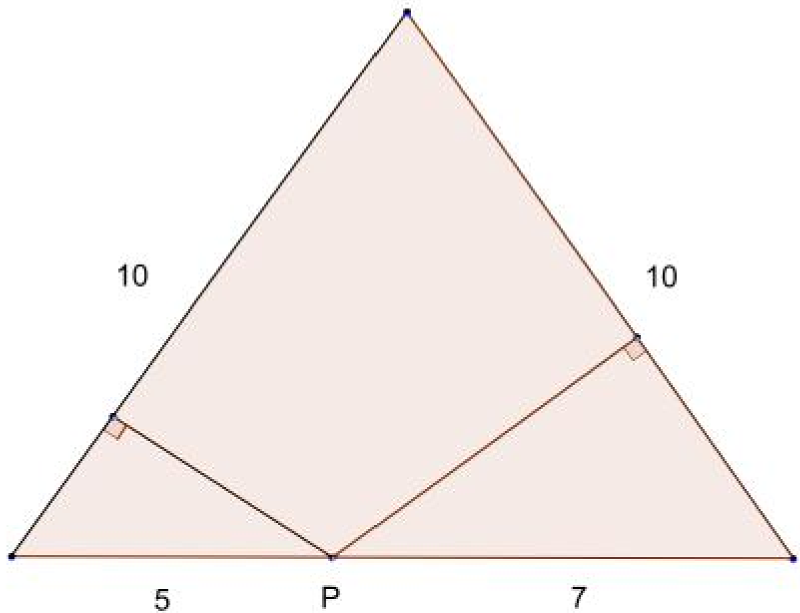
If the triangle with vertices $(1, 0), (3, 3)$, and $(5, 0)$ is revolved 360° about the line $x = 0$. Then, let

$C =$ The area of the triangle.
$D =$ The volume of the solid formed when the triangle is revolved about the line.

Find $A + \frac{B+D}{C\pi}$.

QUESTION 12

An isosceles triangle has legs of length 10 and a base of length 12. A point P is on the base of the triangle such that it is a distance of 5 from one endpoint of the base and 7 from the other. Find the sum of the distances from P to each of the legs of the triangle.



QUESTION 13

Right triangle MNO has hypotenuse MN. Let P be the point on MN such that OP is perpendicular to MN and let Q be the point on ON such that PQ is perpendicular to ON. If $OQ = 4$ and $QN = 9$, find the area of triangle MNO.

QUESTION 14

Let

$A = \sin(45^\circ)$
$B = \tan(30^\circ)$
$C = \cos(60^\circ)$

Find ABC.