Select (E) NOTA *if none of the above answers are correct. Good luck!*

1. Evaluate the sum of the infinite geometric series

$$1 + \frac{2}{3} + \frac{4}{9} + \cdots$$

(A) 3 (B) 2 (C) 4 (D) 6 (E) NOTA

2. Evaluate $f(4)$, given that $f(x) = 5x^3 + 3x + 71$.

(A) 408 (B) 403 (C) 1630 (D) 468 (E) NOTA

3. How many real solutions are there to the equation $x^4 = 3$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

4. Evaluate the sum of the series

$$\sum_{k=1}^{10} (2k)$$

(A) 55 (B) 385 (C) 110 (D) 20 (E) NOTA

5. If the polynomial $2x^3 - x^2 - 5x - 2$ can be expressed in the form $(ax + b)(cx + d)(ex + f)$, where a, b, c, d, e, f are real numbers, find the value of $a + b + c + d + e + f$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

6. Find the coefficient of the x^3 term in the binomial expansion of $(2x^3 + \frac{1}{x})^5$.

(A) 80 (B) 40 (C) 32 (D) 10 (E) NOTA

7. It takes Alex 24 hours to solve an Olympiad problem. It takes Vivek 1 hour to solve an Olympiad problem. Alex works on an Olympiad problem for 4 hours, and then Vivek arrives to assist, and they finish the problem together. How long does it take them to solve the remainder of the problem together once Vivek arrives? Answers are expressed in **minutes.**

(A) 80 (B) 40 (C) 75 (D) 48 (E) NOTA

8. What type of conic section is the following equation?

$$9x^2 + 9y^2 - 18x - 135 = 0$$

(A) Circle (B) Non-circular ellipse (C) Hyperbola (D) Parabola (E) NOTA

9. Consider a regular fair die in the shape of an octahedron, labeled 1 to 8. If it is rolled twice, what is the probability that the product of the numbers obtained is prime?

(A) $\frac{5}{64}$ (B) $\frac{1}{16}$ (C) $\frac{1}{8}$ (D) $\frac{9}{64}$ (E) NOTA

10. The number 2460 has how many positive integer divisors?

(A) 36 (B) 24 (C) 2 (D) 12 (E) NOTA

11. Let $a = \log 3, b = \log 5, c = \log 2$. Express $\log \dfrac{135}{4}$ in terms of $a, b,$ and c.

(A) $3a + b - 2c$ (B) $2(a + b - c)$ (C) $2a - b + 2c$ (D) $a + 2b + c$ (E) NOTA

12. Find the inverse of the function $f(x) = \sqrt{x+5}$.

(A) $f^{-1}(x) = \sqrt{x} + 5$ for $x \geq 0$
(B) $f^{-1} = x^2 + 5$ for $x \geq 0$
(C) $f(y) = \sqrt{y+5}$
(D) $f^{-1}(x) = x^2 - 5$ for $x \geq 0$

(E) NOTA

13. Find the shortest distance between the circle whose equation is $x^2 + y^2 = 16$ and the line $y = -x + 10$.

(A) $10\sqrt{2}$ (B) $4 - 5\sqrt{2}$ (C) $5\sqrt{2}$ (D) $5\sqrt{2} - 4$ (E) NOTA

14. Let the minimum value of the function $y = 2x^2 - 4x + 5$ be a and let the maximum value of the function $y = -x^2 + 2x - 2$ be b. Find $|b - a|$.

(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

15. If $a + b = 2$ and $a^2 + b^2 = 19$, evaluate $a^4 + b^4$.

(A) $\frac{1103}{2}$ (B) 361 (C) $\frac{947}{2}$ (D) $\frac{497}{2}$ (E) NOTA

16. Tony is quite thirsty. In order to please him, Pamela decides to retrieve water to quench his thirst. However, Kevin, who happens to be a supernatural deity, is watching and decides to have a little fun. Let x be his Mischieviousness Level. He decides to distort her trip by somehow defying the laws of physics and making the total distance of her trip 2^{x+3} meters. Pamela's Freak-Out factor is $\dfrac{2^{x+3}k}{d}$, where k is some constant and d is the distance that Pamela's trip would take if Kevin did not interfere. On a day where Kevin's Mischieviousness Level is 3, Pamela's Freak-Out factor is 40. Given that d is always 8 meters, and that Kevin's Mischieviousness Level today is 6, what is Pamela's Freak-Out factor?

(A) 64 (B) 16 (C) 640 (D) 320 (E) NOTA

17. Solve:

$$\log\left(\frac{x+2}{x}\right) - \log(x) = 1$$

(A) $x = \dfrac{1}{2}$ (B) $x = \dfrac{1}{2}, -\dfrac{2}{5}$ (C) $x = 2$ (D) $x = \sqrt{2}$ (E) NOTA

18. Given the function $f(\frac{x}{2}) = x^3 + 2x + 5$, find the product of all values of a for which $f(2a) = -3$.

(A) $\frac{1}{8}$ (B) $\frac{8}{9}$ (C) 512 (D) $-\frac{1}{8}$ (E) NOTA

19. Consider the matrix

$$A = \begin{pmatrix} 1 & x \\ 2 & 3 \end{pmatrix}$$

such that $\det(A^2) = \det(A)$. Solve for all values of x. (Note: $\det(A)$ denotes the determinant of A)

(A) $x = 1$ (B) $x = 3/2$ (C) $x = 0$ (D) $x = 1/6$ (E) NOTA

20. If $\dfrac{x^2 \cdot \sqrt[4]{x^3}}{\sqrt[3]{x} \cdot \sqrt[6]{x^5}}$ can be simplified to the form $x^a \cdot \sqrt[b]{x^c}$, where $x \neq 0$, find the value of $a+b+c$ if a, b, and c are positive integers.

(A) 20 (B) 12 (C) 31 (D) -4 (E) NOTA

21. Solve: $2^{x^2+1} = 32^{2x-4}$.

(A) $x = 7, 3$ (B) $x = 7$ (C) $x = 20$ (D) $x = 8$ (E) NOTA

22. If the roots of the polynomial $x^2 - 4x + 100$ are α and β, evaluate $\alpha^2 + \beta^2$.

(A) 184 (B) -184 (C) 284 (D) -284 (E) NOTA

23. Evaluate the area bounded on the Cartesian plane by the equation

$$49x^2 - 98x + 16y^2 + 224y + 49 = 0$$

(A) 56π (B) 784π (C) 28π (D) 112π (E) NOTA

24. Let $f(x) = (x-2)^{2011} + (x-2)^{2009} + (x-2)^{2007} + \ldots + (x-2)$. Compute the sum of the roots of $f(x)$.

(A) 2011 (B) -2011 (C) 4022 (D) -4022 (E) NOTA

25. How many digits are in the decimal expansion of $4^{20}125^{11}$?

(A) 37 (B) 36 (C) 35 (D) 33 (E) NOTA

26. If $x_1, x_2, x_3, \ldots, x_9$ are the roots of the ninth-degree Kang function $k(x) = 3x^9 + a_8x^8 + a_7x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$, where a_i are real numbers, find the value of a_8 when:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 1 \\ x_2 + x_3 + x_4 + x_5 = 2 \\ x_3 + x_4 + x_5 + x_6 = 3 \\ x_4 + x_5 + x_6 + x_7 = 3 \\ x_5 + x_6 + x_7 + x_8 = 14 \\ x_6 + x_7 + x_8 + x_9 = -5 \\ x_7 + x_8 + x_9 + x_1 = -16 \\ x_8 + x_9 + x_1 + x_2 = 10 \\ x_9 + x_1 + x_2 + x_3 = 8 \end{cases}$$

(A) -20 (B) 15 (C) -15 (D) 30 (E) NOTA

27. There exists an integer k such that $(a+b+c)^3 - a^3 - b^3 - c^3 = k(a+b)(b+c)(c+a)$ for all $a, b, c \in \mathbb{R}$. What is k?

(A) 4 (B) 3 (C) 2 (D) 1 (E) NOTA

28. Let r and s be the roots of the quadratic polynomial $x^2 + 7x + 24$. If $\text{Im}(r) > \text{Im}(s)$, find $r^2 + 8r + s + rs + 24$, where $\text{Im}(z)$ is defined as the imaginary part of z.

(A) 17 (B) 24 (C) $\frac{16-3\sqrt{47}+i(14\sqrt{47})}{4}$ (D) $-30 + 14i\sqrt{47}$ (E) NOTA

29. Evaluate the sum

$$\sum_{j=1}^{10}\sum_{i=1}^{j}(i \cdot j)$$

(A) 1705 (B) 2000 (C) 3025 (D) 3410 (E) NOTA

30. Let $f_n(x)$ be a family of functions defined for positive integers n and x such that $f_n(x) = \frac{1}{n} \cdot \binom{n}{x}$ where $0 \leq x \leq n$. Let a function $f_n(x)$ be denoted as *universally divisible* if $f_n(x)$ is an integer for all integer $x \in [1, n-1]$. Which of the following functions are *universally divisible*?

I. $f_3(x)$ II. $f_7(x)$ III. $f_{77}(x)$ IV. $f_{17}(x)$ V. $f_{186}(x)$

(A) I, II only (B) I, II, III, IV only (C) I, II, IV only (D) I, II, IV, V only (E) NOTA