Pre-Calculus Individual Solutions

1. By the Pythagorean Identity, $\sin^2(x)+\cos^2(x)=1$, B

2. Determinant of an upper and lower triangular matrix is the product of the main diagonal, so $(2)(3)(4)=24$, C

3. After simplifying, we have the equation $x^2+2x-3=0$, factoring gives us $(x+3)(x-1)=0$ giving the solutions -3 and 1, however -3 is an extraneous root thus 1 is the only solution. B

4. If 28 questions are answered correctly, you may only miss 2, so a score of 109 cannot be achieved. D

5. Dot product results in $-4+32-28 = 0$, thus the product is $<0,0,0>$, E

6. As n approaches infinity, the shape approaches a circle (infinite sides), with apothem(radius) of 1, thus the area is pi, D

7. Substituting in 3 results in $0/41$, so the limit is 0. A

8. Sum of the roots is $–b/a$, thus $2/1=2$, D

9. This is the well known Pythagorean Theorem, D

10. $A=\sin(x)$, $R=-\sin(x)$, $\sin(x)/(1+\sin(x))=1/3$, this gives $\sin(x)=1/2$, thus the only answer possible that fits in the given range is $x=\pi/6$, A

11. Using the property $\log_a(b)=\log b/\log a$, this simplifies to $\log_2(2009)$, C

12. Take the first four terms from the right, $(-i^4+i^3-i^2+i)$, with a sum of 0. Every 4 terms is equivalent to this, and this sum is 0, thus everything cancels except i^{2009} which is i, E

13. Substituting in 2 to the polynomial gives us the remainder of 59, A

14. $e^{(-\pi/2)i}=\text{cis}(-\pi/2)=\cos(-\pi/2)+i\sin(-\pi/2)=0-i=-i$, C

15. Multiplying the denominators, we get that $A+B+C=0$, thus $3(A+B+C)=0$, A

16. By the cosine double angle formula, this is equivalent to $\cos(30)$, which is $\sqrt{3}/2$, D

17. 1/2+sqrt{3}/2+sqrt{3}+2sqrt{3}/3+sqrt{3}/3=(5sqrt{3}+5)/2, B

18. Amplitude=|-6|=6, period=2pi/1=2pi, thus sum is 6+2pi, D

19. absin(theta)/2=(10)(60)(sin150)/2=300sin150=150,E

20. B^2-4AC=-80<0, thus this is an ellipse,B

21. cot(2theta)=(A-C)/B, cot(2theta)=1/sqrt{3}, theta=pi/6, A

22. P(Prime with first die)=1/2, now we must consider all the possibilities that add up to a prime with the prime given in the first case. If the prime given we rolled was 2, then the only numbers that would also make the sum a prime would be 1,3 and 5. If the prime rolled from the first die was a 3, then the only numbers would be 2 and 4. The last case is if the prime was a 5, making the only possible numbers be either 2, or 6. Thus from the 2nd die, we have a total of 7 possibilities, out of 18, and multiplying that from the 1/2 from the first die gives us a probability of 7/36, A

23. The information given in the problem suggests a parabola with vertex (0,40) and the two points on the parabola (-25,0) and (25,0). It is easy from here to obtain the equation of the parabola: y=(-8/125)x^2+40. 20 ft from the side is 5 ft from the center, so plugging in 5 results in the height of 192/5, C

24. Using long division results in n^2-100n+10000-999900/(n+100), thus we want 999900/(n+100) to be an integer, and the largest possible value of n that achieves this is when n+100=999900, so n=999800, E

25. Using law of cosines with the angle between the side lengths 5 and 4, results in cos(a)=1/2, there sin(a)=sqrt{3}/2, and the area of the triangle is given by absin(a)/2, or 5sqrt{3}, B

26. The numerator simplifies to 1/sinx, whereas the denominator is an application of sin addition rule resulting in sinx, therefore the answer is csc^2(x), C

27. Since we want this function to be continuous, the two pieces must be equal at k, so we have k^2-k=k^3+k^2-3k, which results in k^3=2k, and since k>0, we get that k=sqrt{2}, E

28. We have infinite geometric series within each other. The first row starts off with 1, 1/2,1/4…, the second row goes 1/2, 1/4, 1/8,…the third row goes 1/4, 1/8, 1/16….etc. The sum of the first row is 1/(1/2)=2, the sum of the second row is (1/2)/(1/2)=1, the sum of the third row is (1/4)/(1/2)=1/2, and so on. Therefore we now have to sum up 2+1+1/2+…, which is given by 2/(1/2)=4, B

29. From the first inequality, we get that $x<\sqrt{y}$, but from the second inequality we have that $x>(y)^{1/4}$, putting these two together, $(y)^{1/4}<x<\sqrt{y}$, raising everything to the fourth power results in $y<x^4<y^2$. When $y=25$, we have $25<x^4<625$, and the only fourth powers between these two are 3 and 4, resulting in the sum of 7, B

30. $[(1+1/4n)^{4n}]1/4=e^{1/4}$, B