Pre-calculus Team Round Solutions

1. p=(1+i)^2008=(2i)^1004=2^1004
   q=(1-i)^2008=(-2i)^1004=2^1004

   A. p+q=2^1004+2^1004=2^1005
   B. p-q=2^1004-2^1004=0
   C. pq=(2^1004)(2^1004)=2^2008
   D. p/q=(2^1004)/(2^1004)=1

2. A) 0
   B) (x^2-x+2)/(x-2)=(x-2)(x+1)/(x-2)=x+1, 2+1=3.
   C) Does not exist/DNE because LHS limit does not equal RHS limit
   D) 1. As exponent approaches 0, anything to the 0-power is 1.

3. f(x)=sqrt{sin2x/2}, g(x)=3sinx+4cosx
   A) max value of sin is 1, therefore max value of this is sqrt{2}/2
   B) Since this is square root of something, we can't take negative values, thus the min value is 0
   C) Amplitude of g(x) is sqrt{3^2+4^2}=5, thus max value is 5
   D) Min value is then -5

4. A) Dot product: 1(5)+0(0)+5(1)=10
   B) Magnitude=sqrt{1^2+0^2+5^2}=sqrt{26}
   C) doing 3x3 matrix with <i,j,k> in the first row, and the two vectors in the next, the result is the cross product: <0,24,0>
   D) A-B=<1-5,0-0,5-1>=<-4,0,4>

5. A) sinxcosx=sin2x/2, x=30, sin60/2=sqrt{3}/4

   B) 3sinx-4sin^3x=sin3x, x=10, sin30=1/2

   C) sec^x-1=tan^2x, x=pi/4, tan^2(pi/4)=1

   D) sqrt{(1-cosx)/(1+cosx)}=tan(x/2),x=30, tan 15=7-4sqrt{3}

6. A=5, B=2pi/pi=2, C=Horizontal Phase Shift=-C/B=4/pi, D=3

7. f(x)=[(x)(x+1)/2]^x

A) f(1)=1^1=1

B) f(2)=3^2=9

C) As x→infinity, f(x)→infinity or DNE

D) Note that inverse of h(x)=inverse of f(x), thus [(x)(x+1)/2]^x=216, x=3

8. A) 1/log_75(x)=log_x(75), 1/log_3(x)=log_x(3), log_x(25)=2, x=5

B) 5x-1=In2, x=(In2+1)/5

C) 6^(log_6(5))=5, 3^(log_27(7))=7^(1/3), 2^(log_8(9))=9^(1/3), 5+7^(1/3+9^(1/3)

D) In(2^x)/In(5^x)=log_5(2), which is not equivalent to 25/2, thus no solution.

9. A) t=0, x(0)=4, y(0)=1, Waldo is at point (4,1)

B) t^2+1=2t+4, t^2-2t-3=0, (t-3)(t+1)=0, t>0, thus t=3

C) 2t+4=10, t=3, y(3)=10

D) t=(x-4)/2, y=(x-4)^2/4+1→y-1=(x-4)^2/4, this is a general form for a parabola

10. Conic is an ellipse in the form: (x-1)^2/9+(y+1)^2/4=1,

A) Area is (3)(2)(pi)=6pi

B) Major axis=6, minor axis=4, sum is 10

C) Latus rectum=2b^2/a→ 2(4)/3=8/3

D) Sum of first 50 WHOLE numbers is 0+1+2+3…+49=49(50)/2=49(25)=1225

11. A) This is the Fibonacci Sequence where the next term is provided by the sum of the previous two terms

B) The next 3 terms are 13, 21 and 34, thus the sum of these numbers is 68

C) Let F(n) represent the nth Fibonacci number; thus this sequence is F(n+1)/F(n). The problem is asking the limit as n→infinity of F(n+1)/F(n). This well known ratio is the golden ratio, or (1+sqrt(5))/2

D) f(n) is the formula for the nth term of the Fibonacci numbers, thus f(6)=8.

12. A) Good numbers are perfect squares, thus between 0 and 49^2 exclusive, we have 1-48, or 48 good numbers.

B) Cool numbers are numbers that are NOT perfect squares. 49^2=2401, and excluding 0 and 2401, we have 2400 numbers in this interval. Since we counted 48 good numbers in this set, we have 2400-48=2352 cool numbers.

C) Good numbers are perfect squares, thus k=2

D) It is easy to see that as n grows larger, the number of perfect squares grows smaller compared to the rest of the numbers, thus the limit as n approaches infinity is 0.