

The choice E. NOTA means that none of the other answers are correct. Good luck!

1. If  $x = \sum_{i=1}^{\infty} \frac{i^4 + 3i^3 + 1337}{i^{1337}}$ , what is the value of  $\sin^2 x + \cos^2 x$ ?  
A. 0                      B. 1                      C. 1337                      D. 1787569                      E. NOTA
2. Find the determinant of the matrix  $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ .  
A. 6                      B. 12                      C. 24                      D. 48                      E. NOTA
3. Find the sum of the solutions to the equation  $\log_3(x + 2) + \log_3(x) = 1$ .  
A. -2                      B. 1                      C. 2                      D. 3                      E. NOTA
4. Dr. Fraser gave a 30 question pop quiz on which each correct answer was worth 4 points, a blank answer was worth 0 points, and each incorrect answer was worth -1 points. What is the least positive integral score that was not attainable on this quiz?  
A. 103                      B. 105                      C. 107                      D. 109                      E. NOTA
5. Let  $A = \langle -1, 4, 7 \rangle$  and  $B = \langle 4, 8, -4 \rangle$ . What is the product of  $A \bullet B$  and  $A \times B$ ?  
A.  $\langle -72, 24, -24 \rangle$     B.  $\langle 72, 24, 24 \rangle$         C.  $\langle 72, -24, -24 \rangle$     D.  $\langle -72, -24, 24 \rangle$     E. NOTA
6. Let the area of a regular  $n$ -gon with apothem of length 1 be  $A(n)$ . What is  $\lim_{n \rightarrow \infty} A(n)$ ?  
A. 1                      B.  $\sqrt{2}$                       C.  $\sqrt{3}$                       D.  $\pi$                       E. NOTA
7. Evaluate  $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 + 7x + 12}$ .  
A. 0                      B.  $\frac{4}{13}$                       C. 1                      D. DNE                      E. NOTA
8. Find the sum of the roots of the equation  $x^3 - 2x^2 + 1 = 0$ .  
A. -2                      B. -1                      C. 1                      D. 2                      E. NOTA
9. In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. This is known as the \_\_\_\_\_ Theorem.  
A. Euclidean            B. Fundamental        C. Geometric            D. Pythagorean        E. NOTA

10. The sum of the infinite geometric series  $\sin x - \sin^2 x + \cdots$  is  $\frac{1}{3}$ . Find  $x$ , given that  $0 \leq x \leq \frac{\pi}{2}$ .
- A.  $\frac{\pi}{6}$                       B.  $\frac{\pi}{4}$                       C.  $\frac{\pi}{3}$                       D.  $\frac{\pi}{2}$                       E. NOTA
11. Evaluate  $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdots \log_{2008} 2009$ .
- A. 1                      B. 11                      C.  $\log_2 2009$                       D.  $\frac{2019045}{2008!}$                       E. NOTA
12. Evaluate  $i^{2009} - i^{2008} + i^{2007} - i^{2006} + \cdots + i^3 - i^2 + i$ , where  $i = \sqrt{-1}$ .
- A. 0                      B. 1                      C.  $1 + i$                       D.  $-1 - i$                       E. NOTA
13. Find the remainder when  $(2x^{2009} - 4x^{2008} + 8x^{2001} - 16x^{2000} + 32x - 5)$  is divided by  $(x - 2)$ .
- A. 59                      B. 62                      C. 65                      D. 69                      E. NOTA
14. Given that  $\ln(\cos \theta + i \sin \theta) = i\theta$ , evaluate  $e^{-\frac{\pi}{2}i}$ . Note that  $i = \sqrt{-1}$ .
- A.  $i$                       B.  $-1$                       C.  $-i$                       D. 1                      E. NOTA
15. When  $\frac{4x + 7}{x^3 + 3x^2 - 4x}$  is written as  $\frac{A}{x} + \frac{B}{x + 4} + \frac{C}{x - 1}$ , what is  $3(A + B + C)$ ?
- A. 0                      B. 1                      C. 2                      D. 3                      E. NOTA
16. Evaluate  $2 \cos^2 15^\circ - 1$ .
- A. 0                      B.  $\frac{1}{2}$                       C.  $\frac{\sqrt{2}}{2}$                       D.  $\frac{\sqrt{3}}{2}$                       E. NOTA
17. Evaluate  $\cos 60^\circ + \sin 60^\circ + \tan 60^\circ + \sec 60^\circ + \csc 60^\circ + \tan 60^\circ$ .
- A. 5                      B.  $\frac{5\sqrt{3}+5}{2}$                       C.  $5\sqrt{3}$                       D.  $5\sqrt{3} + 5$                       E. NOTA
18. Consider the function  $f(\theta) = 1337 - 6 \sin(\theta - \frac{\pi}{4})$ . Find the sum of the amplitude and period of  $f$ .
- A.  $-14$                       B.  $-2$                       C.  $-6 + 2\pi$                       D.  $6 + 2\pi$                       E. NOTA
19. What is the area of a triangle with two sides of length 10 and 60 that meet at a  $150^\circ$  angle?
- A. 300                      B.  $300\sqrt{3}$                       C. 600                      D.  $600\sqrt{3}$                       E. NOTA

For questions 20-21, consider the conic  $8x^2 + 4\sqrt{3}xy + 4y^2 + 4x + 8y = 256$ .

20. Given that the conic is not degenerate, what type of conic is it?

- A. Circle                  B. Ellipse                  C. Hyperbola                  D. Parabola                  E. NOTA

21. The angle of rotation,  $0 \leq \theta \leq \frac{\pi}{2}$ , for this conic is:

- A.  $\frac{\pi}{6}$                   B.  $\frac{\pi}{4}$                   C.  $\frac{\pi}{3}$                   D.  $\frac{\pi}{2}$                   E. NOTA

22. Dhyan, who loves Statistics, has two dice. He rolls the first die, and if the number is *not* prime, he records this number. If he rolls a prime, he then rolls the second die and records the sum of the two numbers rolled (on the first and second die). What is the probability that the number Dhyan records is prime?

- A.  $\frac{7}{36}$                   B.  $\frac{9}{36}$                   C.  $\frac{11}{36}$                   D.  $\frac{17}{36}$                   E. NOTA

23. Eli notices that Pratik's parabolic house is symmetric about the middle of the house, and has a maximum height of 40 feet. If the house is 50 feet wide, how many feet tall is it 20 feet from either side?

- A.  $\frac{86}{5}$                   B.  $\frac{191}{5}$                   C.  $\frac{192}{5}$                   D.  $\frac{202}{5}$                   E. NOTA

24. What is the largest positive integer  $n$  for which  $n^3 + 100$  is divisible by  $n + 100$ ?

- A. 0                  B. 1                  C. 320                  D. 485                  E. NOTA

25. Find the area of a triangle with side lengths 5, 4, and  $\sqrt{21}$ .

- A.  $\frac{5\sqrt{3}}{2}$                   B.  $5\sqrt{3}$                   C.  $10\sqrt{3}$                   D.  $20\sqrt{3}$                   E. NOTA

26. Simplify  $\frac{\csc x \cdot \sec x \cdot \cot x \cdot \sin x}{\sin 3x \cos 2x - \cos 3x \sin 2x}$ .

- A.  $\cot^2 x$                   B.  $\tan^2 x$                   C.  $\csc^2 x$                   D.  $\sec^2 x$                   E. NOTA

27. Find the value of  $k > 0$  so that  $f$  is continuous.

$$f(x) = \begin{cases} x^2 - x & x < k \\ x^3 + x^2 - 3x & x \geq k \end{cases}$$

- A. 0                  B. 1                  C. 2                  D. 3                  E. NOTA

28. Let  $A_\infty$  be an infinite two-dimensional array (infinite number of rows and columns). If  $A_{ij}$ , the element in row  $i$  and column  $j$  of  $A_\infty$ , is  $2^{(2-i-j)}$ , what is the sum of all elements in the array?

- A. 2                      B. 4                      C. 8                      D.  $\infty$                       E. NOTA

29. Consider the inequalities:

$$\log_2 x < \log_4 y \quad \text{and} \quad \log_3 x > \log_{81} y$$

If  $y = 25$  and  $x > 1$ , find the sum of all possible integer values for  $x$ .

- A. 5                      B. 7                      C. 9                      D. 194675                      E. NOTA

30. Given that  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$ , what is  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{4n}\right)^n$ ?

- A.  $e^{\frac{1}{8}}$                       B.  $e^{\frac{1}{4}}$                       C.  $e^{\frac{1}{2}}$                       D.  $e$                       E. NOTA