## 2009 James S. Rickards Fall Invitational

1. (B) The probability between flips is independent. There is always a 50% chance of obtaining a tail on any given flip of a fair coin.

2. (C) Since 3 cosairs are worth 4 overlords, 9 cosairs are worth 12 overlords. 3 overlords are worth 4 hydralisks, so 12 overlords are worth  $4 \times 4=16$  hydralisks.

3. (D) This is a definition, part of Vietes formulae. Looking at a linear and quadratic equation will also arrive at this result.

4. (C) I. If you do not live in Montana, you do not live in the United States. Falseyou live in Florida (not Montana), yet you live in the United States. II. If you live in the United States, you live in Montana. Falseyou live in the United States, but do not live in Montana. III. The contrapositive of a true statement is always true. You cannot live outside of the United States, but inside of Montana.

5. (C) Let  $\sqrt{5-2\sqrt{6}} = \sqrt{a} - \sqrt{b}$ . Squaring both sides gives  $5-2\sqrt{6} = a+b-2\sqrt{ab}$ . Thus, a+b=5 and ab = 6, which has solutions 3, 2 and 2, 3. However, the value must be positive, so a = 3 and  $b = 2 \rightarrow \sqrt{3} - \sqrt{2}$ .

6. (C) By casework or the Chicken McNugget formula, the answer is  $5 \times 11 - 5 - 11 = 39$ .

7. (B) We need all integers such that 7x + 2 is divisible by 3. Note that 7x + 2 = 6x + (x + 2), and 6x is divisible by 3. Therefore, x + 2 must be divisible by 3. Thus,  $x = \cdots - 5, -2, 1, 4, 7 \cdots$  and can be expressed in the form 3n + 1. 3 + 1 = 4.

8. (C) Let  $x = \sqrt{2 + \sqrt{2 + \cdots}}$ . Then,  $x^2 = 2 + x$ , so x = 2 or -1. But, x > 0, so x = 2.

9. (D) Note that  $\log 45 = \log(5*9) = \log(10/2) + \log(3^2) = \log 10 - \log 2 + 2\log 3 = 1 - M + 2N$ .

10. (B) From the given equations, we can construct the inequality a > c > d > b. But note that these are the times taken the winner takes the least time  $\rightarrow$  b, Blake.

11. (B) If k is the answer,  $1 + \sqrt{2k+1} = k$ , or  $\sqrt{2k+1} = k-1$ . Squaring both sides gives  $2k+1 = k^2 - 2k + 1$ , so  $2k = k^2 - 2k$ , or  $k^2 = 4k \rightarrow k = 0$  or 4. But, k = 0 is extraneous.

12. (E) When n is odd, the sum is 0. When n is even, the sum is 1. None of the given expressions are equivalent to this.

13. (C) For four years, (x,y) could be (1,3), (2,2) or (3,1). There are 5 choices for the first school attended, and 4 left for the second school. 3 \* 5 \* 4 = 60.

14. (C)  $a^6 - a^4b^2 + 4a^2b^4 - 4b^6 = a^4(a^2 - b^2) + 4b^4(a^2 - b^2) = (a^4 + 4b^4)(a^2 - b^2) = (a^2 + 2ab + 2b^2)(a^2 - 2ab + 2b^2)(a + b)(a - b) \to 4.$ 

15. (B) Converting to base 10, we have  $33_4 = 3(4^0) + 3(4^1) = 15$  and  $53_6 = 3(6^0) + 5(6^1) = 33$ . Conveniently, 15+33=48, so b=10.

16. (A) Minutes = k(carrots)/(bunnies). There are 3/2 as many bunnies, and only 3/4 as many carrots. Thus, the number of minutes it will take is 16(2/3)(3/4)=8.

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17. (C) Note that medians split a triangle into six triangles of equal area. Also, note that triangle ABF, which consists of 3 of these triangles + [BEF], has area equal to triangle ABC since they have the same base and height. [BEF]=[ABF] 3 of these triangles = 6 of these triangles 3 of these triangles = 3 triangles. Thus, BEF has area half of ABC. Triangle BDE is one of the six triangles. The ratio is 1:3.

18. (B) The sum of the roots is -b/a, but there is no  $x^4$  term, so b=0. -0/3=0.

19. (B)  $\ln(e^x) = x = 3 = \ln(e^3).$ 

20. (B) Let x=2000, y=9, and x+y=2009. Then: 
$$\frac{x^3+3x^2y+3xy^2+y^3-x^3-y^3}{(x+y)xy} = 3\frac{x^2y+xy^2}{x^2y+xy^2} = 3.$$

21. (C) Surface area =  $4(\pi)(r^2) \rightarrow r^2 = 9/\pi \rightarrow r = 3/\sqrt{\pi}$ . Volume=  $4(\pi)(r^3)/3 = 4(\pi)(27/((\pi)\sqrt{\pi})/3) = 36/\sqrt{\pi}$ .

22. (C) The probability that the numbers are equal is 1/10. If they are not equal, one is equally likely to be greater than the other. Thus, we have (1-1/10)/2=9/20.

23. (E) One quick method is Newtons sums. Note  $(\text{sum of roots})^2 = (\text{sum of squares of roots}) + 2(\text{product of roots taken two at a time})$ . Thus our desired value is 16-2(-5)=26.

24. (A) Let this sum be S. Then, 3S=3+2+3/3+4/9+5/27+ Subtracting S from 3S, we have 2S=3+1+1/3+1/9+=3+1.5=4.5. Thus, S=(9/2)/2=9/4.

25. (C) 
$$f(0) = (\pi^0)/(0!) = 1/1 = 1.$$

26. (D) This is the Maclaurin series expansion of  $e^n$  for  $n = \pi$ .  $\ln(e^{\pi}) = \pi$ .  $[[\pi]] = 3$ .

27. (D) Draw a rough graph of the two functions, and they will intersect three times.

28. (E)  $C=A \to 2(\pi)(r) = (\pi)(r^2) \to 2 = r$ . The diameter is 2r = 4.

29. (D) Note that  $\sin(90-x) = \cos x$ . Construct a right triangle with an angle x. Then, if  $\cot(x) = a$ , the triangle has hypotenuse  $\sqrt{a^2 + 1}$ . Thus,  $\cos(x) = a/\sqrt{a^2 + 1}$ .

30. (D) The phrase is spelled by getting a letter from the front, then the end, then the front, etc. Using this, you should have "the answer to this question is d".