QUESTION 1

Let

$$
\begin{aligned}
A &= \text{the distance between the point } \left(5, \frac{7}{2}\right) \text{ and the center of the circle } 4x^2 - 32x + 4y^2 - 12y + 53 = 0. \\
B &= \text{the length of the latus rectum of the parabola } x^2 + 6x - 6y + 33 = 0. \\
C &= \text{the eccentricity of the hyperbola } 6x^2 + 24x - 8y^2 + 48y - 120 = 0. \\
D &= \text{the sum of the lengths major and minor axes of the conic section } \frac{(x-2)^2}{9} + \frac{(y+4)^2}{4} = 1.
\end{aligned}
$$

Compute the value of $\dfrac{D}{A^2} + BC^2$.

QUESTION 2

Let

$$\begin{aligned} A_{10} &= 432_7. \\ B_8 &= 2012_4. \\ C &= \text{the number of zeroes in the binary representation of } 4097_{10}. \end{aligned}$$

The base 8 number aba_8 where a, b are digits has a two digit base 10 representation which is divisible by 9. Let $D = a + b$.

Compute the value of $CD - (A - B)$.

QUESTION 3

Let

$$
\begin{aligned}
A &= \operatorname{cis}\left(\frac{2\pi}{21}\right)^7 + \operatorname{cis}\left(\frac{5\pi}{24}\right)^8 + \operatorname{cis}\left(\frac{11\pi}{54}\right)^9 + \operatorname{cis}\left(\frac{7\pi}{60}\right)^{10} \\
&\quad \text{given that } \operatorname{cis}(\theta)^n = \operatorname{cis}(n\theta) \text{ and } \operatorname{cis}(\theta) = \cos\theta + i\sin\theta. \\
B &= a+b+c+d+e+f \text{ where } x^6 - 1 = (ax^5 + bx^4 + cx^3 + dx^2 + ex + f)(x-1) \\
&\quad \text{and } a, b, c, d, e, f \text{ are all integers.} \\
C &= \frac{(1+i\sqrt{3})^6}{(1+i)^8}. \\
D &= \left|\frac{5+i\sqrt{11}}{3+3i}\right|.
\end{aligned}
$$

Compute the value of $B - D^C - A$.

QUESTION 4

Let

$$
\begin{aligned}
A &= \text{the sum of the first 20 terms in the sequence } \{0, -3, 1, -4, 2, -5, 3, -6, \ldots\}. \\
B &= \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots. \qquad \text{Note: The general term is } \frac{n}{2^n}. \\
C &= \sin(x), \text{ where } \sin(x) - \sin^2(x) - \sin^3(x) + \sin^4(x) - \cdots = \frac{1}{3} \text{ and } x \text{ is in the interval } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].
\end{aligned}
$$

The terms $a_1, a_2, a_3, \ldots$ form an arithmetic sequence with common difference $d > 0$. Furthermore, the terms a_1, a_5, a_{10} form a geometric sequence with a common ratio $r > 0$. Let $D = d$ if $a_1 = 5$.

Compute the value of $\dfrac{AC}{BD}$.

QUESTION 5

Let

$$A = \left[\left[\left(\frac{2011}{2010}\right)^{2010}\right]\right], \text{ given that } \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e \text{ and } [[x]] \text{ denotes the greatest integer less than or equal to } x.$$

$$B = x, \text{ where } x \text{ is real and } \frac{e^{3x}-2e^{2x}-5e^x+6}{e^x-3} = 0.$$

$$C = y, \text{ where } i^i = e^x, \ x \text{ is in the interval } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \text{ and } \sqrt{-1} = i.$$

$$D = e^{\frac{5\pi i}{2}} \text{ when written in the form } \cos\theta + i\sin\theta.$$

Compute the value of $A + Bi + C + Di$.

QUESTION 6

Let

$$\begin{aligned}
A &= \begin{vmatrix} \sin\theta & \cos^2\theta \\ 1 & -\sin\theta \end{vmatrix}. \\
B &= \text{the sum of the solutions to } \begin{vmatrix} x+1 & 1 \\ 2x+6 & x \end{vmatrix} = 0. \\
C &= \begin{vmatrix} 3 & -\frac{5}{2} & 0 \\ 4 & -6 & -1 \\ 8 & -7 & 2 \end{vmatrix}. \\
D &= \text{the area of triangle with vertices } (1,3),\ (7,-5),\ \text{and } (4,0).
\end{aligned}$$

Compute the value of $A+B+C+D$.

QUESTION 7

Let

$$\begin{aligned} A &= \text{the magnitude of the cross product of the vectors } \langle 2,0,1\rangle \text{ and } \langle 3,-1,4\rangle\,. \\ B &= \langle 2,-5,3\rangle \cdot \langle 6,0,1\rangle\,. \\ C &= x,\ \text{where the vectors } \left\langle \frac{5}{3}, -\frac{2}{15}\right\rangle \text{ and } \left\langle \frac{9}{10}, x\right\rangle \text{ are orthogonal.} \\ D &= \text{the magnitude of the vector } \langle 4,-5,3\rangle\,. \end{aligned}$$

Compute the value of $\dfrac{(AD)^2}{BC}$.

QUESTION 8

Let

$$\begin{aligned} A &= \text{the coefficient of the } x^2y^3 \text{ term in the expansion of } (3x-2y)^5. \\ B &= \text{the sum of the coefficients in the expansion of } (7x-9y)^8. \\ C &= (1-i)^{12}. \\ D &= (a-p)(b-p)(c-p)\cdots(y-p)(z-p). \end{aligned}$$

Compute the value of $\dfrac{B}{C} + A - D$.

QUESTION 9

With respect to the sinusoidal function $f(x) = -4\cos(2x+3)+5$, let

$$\begin{aligned} A &= \text{the amplitude of the function.} \\ B &= \text{the phase shift of the function.} \\ C &= \text{the period of the function.} \\ D &= \text{the vertical shift of the function.} \end{aligned}$$

Compute the value of $ABCD$.

QUESTION 10

Let

$$\begin{aligned} A &= \text{the number of petals of the polar graph } r = 4\sin(3\theta). \\ B &= \text{the eccentricity of the polar graph } r^2 = 1. \end{aligned}$$

The polar point $\left(4, \dfrac{\pi}{6}\right)$ when written in rectangular form is (C, D).

Compute the value of $C^B + \dfrac{A}{D}$.

QUESTION 11

Let

$$\begin{aligned} A &= \tan^2\left(\frac{4\pi}{3}\right). \\ B &= \csc(30^\circ). \\ C &= \sin(2\theta) \text{ where } \cos\theta = \frac{1}{4},\ 0 < \theta < 90^\circ. \\ D &= \sin^2\left(\frac{\pi}{24}\right) + \cos^2\left(\frac{\pi}{24}\right). \end{aligned}$$

Compute the value of $\dfrac{C}{A+B+D}$.

QUESTION 12

Let

$$\begin{aligned} A &= x, \text{ where } 2^{22} + 2^{24} = x \cdot 2^{22}. \\ B &= y, \text{ where } \tan\left(\frac{\pi}{3}\right) = 81^y. \\ C &= z, \text{ where } \log_2 z = \frac{\log_2 5 + \log_2 9}{2}. \\ D &= 100^{2\log 5}. \end{aligned}$$

Compute the value of $\dfrac{(AC)^2}{BD}$.

QUESTION 13

Let

$$\begin{aligned} A &= \text{the area of } \triangle ABC \text{ with } AB = 4,\ AC = 6,\ \text{and } \angle BAC = 30^\circ. \\ B &= \text{the area of a regular octagon with a side length of 2.} \\ C &= \text{the length of diagonal } BD \text{ in quadrilateral } ABCD \text{ with } AB = 4,\ AD = 8,\ \text{and } \angle A = 60^\circ. \\ D &= \text{the area of a triangle with sides of length 3, 4, and 5.} \end{aligned}$$

Compute the value of $\dfrac{C^2 B}{AD}$.

QUESTION 14

Chris Kim just received a huge bag of candy. In the bag, there are 9 peppermints, 6 lollipops, 10 chocolate bars, and 5 jawbreakers. Chris Kim picks two candies at random from the bag without replacement.

$$
\begin{aligned}
A &= \text{the probability that he will pick a peppermint and a jawbreaker.} \\
B &= \text{the probability that both candies are chocolate bars.} \\
C &= \text{the probability that the second candy drawn is a lollipop.}
\end{aligned}
$$

Compute the value of $A + B + C$.