For this test, let $i = \sqrt{-1}$ *and* $\text{cis}\theta = \cos\theta + i\sin\theta$. *Select* (E) NOTA *if none of the above answers are correct. Good luck!*

1. In Boolean Algebra, 1+1=1, 1+0=1, 0+1=1, 0+0=0 and both 0 and 1 are complements of each other ($\bar{0} = 1$, $\bar{1} = 0$). Then, in Boolean Algebra, what is $(\overline{(1+1)} + 1) + (0+1)$?
(A) 1 (B) 0 (C) -1 (D) 2 (E) NOTA

2. Evaluate $\cos 120° + \sin 135° + \tan 315°$.
(A) $-\frac{1}{2} + \frac{\sqrt{2}}{2}$ (B) $\frac{3}{2} + \frac{\sqrt{2}}{2}$ (C) $-\frac{3}{2} + \frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}$ (E) NOTA

3. Evaluate $\begin{vmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$.
(A) 1 (B) $\cos 2\theta$ (C) $-\sin 2\theta$ (D) $-\cos 2\theta$ (E) NOTA

4. Evaluate $\dfrac{\sqrt{1\cdot 5\cdot 10 + 2\cdot 10\cdot 20 + \cdots + n\cdot 5n\cdot 10n}}{\sqrt{1\cdot 9\cdot 25 + 2\cdot 18\cdot 50 + \cdots + n\cdot 9n\cdot 25n}}$ where $n = 2010$.
(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{2}}{15}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\sqrt{5}$ (E) NOTA

5. Convert 3π radians to degrees.
(A) 270° (B) 540° (C) 630° (D) 360° (E) NOTA

6. Given $f(\theta) = \cos^2\theta - \sin^2\theta$, find the value of $f(\frac{\pi}{6})$
(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\sqrt{3}}{2}$ (E) NOTA

7. What is the dot product of the vectors $\langle 1, 3\rangle$ and $\langle 7, 2\rangle$?
(A) -19 (B) 36 (C) 1 (D) 13 (E) NOTA

8. Find the sum of the distinct solutions of $2\cos x + \sin 2x = 0$ on the interval $[0, 2\pi)$.
(A) $\frac{3\pi}{2}$ (B) $\frac{5\pi}{2}$ (C) $\frac{7\pi}{2}$ (D) $\frac{9\pi}{2}$ (E) NOTA

9. If $a+b+c = 1$, $a^2+b^2+c^2 = 5$, and $a^3+b^3+c^3 = 7$, what is $ab(a+b) + bc(b+c) + ca(c+a)$? Note: $\{a, b, c\} \in \mathbb{R}$.
(A) -2 (B) 0 (C) 1 (D) 2 (E) NOTA

10. Compute the value of $\left|\frac{47+31i}{31+47i}\right|$.
(A) $\sqrt{3170}$ (B) $\frac{47}{31}\text{cis}720°$ (C) $\frac{31}{47}\text{cis}720°$ (D) 3170 (E) NOTA

11. Let $x + \frac{1}{x} = 2\cos(13°)$. Then, $x^2 + \frac{1}{x^2} = 2\cos(y°)$, where $0 < y < 90$. Find the sum of the digits of y.
(A) 8 (B) 4 (C) 7 (D) 10 (E) NOTA

12. Let $(1 - i\sqrt{3})^4 = a + bi$. Determine the value of a, given that $\{a, b\} \in \mathbb{R}$.
(A) 8 (B) -8 (C) 16 (D) $16\sqrt{3}$ (E) NOTA

13. Compute $\tan 10° \tan 20° \tan 30° \tan 40° \tan 60° \tan 50° \tan 60° \tan 70° \tan 80°$.

(A) $\frac{1}{2} + \frac{\sqrt{3}}{2}$ (B) 1 (C) $\sqrt{3}$ (D) $\cot 10° \cot 20° \cot 30°$ (E) NOTA

14. The coach of a certain high school basketball team is holding a tryout. The players who make the cut will be assigned jerseys with numbers from 0 to 20 (0, 1, 2, 3, ..., 20). What is the minimum number of players that the team must have to ensure that two consectuive jersey numbers will be assigned?

(A) 10 (B) 11 (C) 12 (D) 13 (E) NOTA

15. Given $f(x) = (x + 1)$ and $g(x) = x^2$, find $f(x) + g(x) + (f \circ g)(x) + (f \cdot g)(x)$

(A) $x^3 + 3x^2 + 3x + 2$ (B) $x^3 + 3x^2 + x + 2$ (C) $x^3 + 2x^2 + 2x + 1$ (D) $x^3 + 3x^2 + x + 1$ (E) NOTA

16. In triangle $\triangle ABC$, $\overline{BC} = 4$, $\overline{AC} = 7$, and $\angle C = 60°$. Find $\overline{AB}$.

(A) $\sqrt{37}$ (B) $\sqrt{93}$ (C) 3 (D) $\triangle$ Does Not Exist (E) NOTA

17. The roots of the polynomial $x^{18} - 1 = 0$ are called the eighteenth roots of unity, namely $r_1, r_2, r_3, \ldots, r_{18}$. Given that $r_1 = 1$, compute the value of $r_2 + r_3 + \cdots + r_{18}$.

(A) 1 (B) $\text{cis}(\frac{\pi}{9})$ (C) $\text{cis}(\frac{\pi}{18})$ (D) -1 (E) NOTA

18. If $\dfrac{5x+7}{x^2+3x+2} = \dfrac{A}{x+1} + \dfrac{B}{x+2}$ where A and B are real numbers, then $A \cdot B =$

(A) 4 (B) 5 (C) 6 (D) 18 (E) NOTA

19. Evaluate $\cos(\frac{\pi}{|a|})$ given that $\begin{cases} 2x - a = 7 \\ x + 2y + 3a = 2 \\ x - y = 2 \end{cases}$.

(A) -1 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) NOTA

20. Steve and Dan are running a 100 meter race (where 100 meters is the distance from the starting line to the finish line). Steve starts 10 meters in front of the starting line, and Dan starts at the starting line. However, Steve was 4 meters behind the finish line when Dan finished the race. How far ahead, in meters, of the starting line would Steve have to start for both Steve and Dan to finish the race together? (They both run at constant rates the whole race).

(A) 12 (B) 14 (C) 16 (D) 18 (E) NOTA

21. Determine the value of the eccentricity of the non-degenerate conic section $x^2 + 2xy + y^2 - 5x + 11y + 130 = 0$.

(A) 0 (B) 1 (C) -1 (D) 2 (E) NOTA

22. Let a equal the sum of the possible remainders when a perfect square is divided by 5 and let b equal the sum of the possible remainders when a perfect square is divided by 6. Compute $5\cos(a\pi) + \sin(b\pi)$.

(A) -5 (B) -1 (C) 0 (D) 1 (E) NOTA

23. Determine the type of conic section in the $xy-$plane given by the parametric equations $y = \sin^2 t$ and $x = 5\cos t$.

(A) circle (B) ellipse (C) hyperbola (D) parabola (E) NOTA

24. Which of the following is/are true?

    I. If two vectors have the same direction, but different magnitudes, then one vector must be a scalar multiple of the other.

    II. If one vector is a scalar multiple of another vector, they must have the same direction.

    III. The product of two singular matrices must be singular, where the product is defined.

    IV. The product of two nonsingular matrices must be nonsingular, where the product is defined.

    (A) I, II, III, IV (B) I only (C) I, II only (D) I, III, IV only (E) NOTA

25. Given that $\log_b(\sin\theta) = a$, express $\dfrac{\cos^2\theta}{1+\sin\theta}$ in terms of a and b, where $0 < \theta < \frac{\pi}{2}$.

    (A) $1-a$ (B) $1-b$ (C) $1-b^a$ (D) $1-a^b$ (E) NOTA

26. Determine the acute angle θ such that $2\sin 50^\circ \sin 70^\circ = \frac{1}{2} + \sin\theta$.

    (A) 60° (B) 70° (C) 80° (D) 85° (E) NOTA

27. The Kang function, given as $k(x) = x^4 + ax^3 + bx^2 + c$, has some strange properties. Both the arithmetic mean and the product of the zeroes of $k(x)$ are equal to the value of $k(1)$. If the y-intercept of the graph $y = k(x)$ is 3, compute the value of abc. Note: $\{a, b, c\} \in \mathbb{R}$.

    (A) -630 (B) -210 (C) 2 (D) 3 (E) NOTA

28. Let θ be the acute angle between the line $x\sqrt{3} - 3y = -1500$ and the x-axis. Compute the value of $\cos^2\theta - \sin^2\theta$.

    (A) $-\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$ (E) NOTA

29. Given that $|x-1|+|y-2| = 0$, compute the value of $\dfrac{1}{xy} + \dfrac{1}{(x+1)(y+1)} + \dfrac{1}{(x+2)(y+2)} + \cdots + \dfrac{1}{(x+1335)(y+1335)}$.

    (A) $\frac{1334}{1335}$ (B) $\frac{1}{1337}$ (C) 0 (D) $\frac{1335}{1337}$ (E) NOTA

30. Triangle ABC has sides $AB = x$, $BC = x$, and $AC = 1$. Solve for $(\cos(\angle B) - 1)(\sec(\angle A))^2$ where $\cos\angle A \neq 0$ and x is a positive integer.

    (A) -2 (B) -1 (C) 1 (D) 2 (E) NOTA