The choice E. NOTA means that none of the other answers are correct. Good luck!

1. Evaluate $\lim_{x\to 2} \dfrac{2x-4}{x^2+x-2}$.

   A. $-\frac{1}{2}$ B. 0 C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. NOTA

2. Evaluate $\lim_{x\to \pi} \dfrac{\sec x - \cos x}{\tan^2 x}$.

   A. -1 B. 0 C. 1 D. does not exist E. NOTA

3. Let I be an interval containing the point a. Let f, g, and h be functions defined on I, except possibly at a itself. Suppose that for every x in I not equal to a, we have $g(x) \le f(x) \le h(x)$, and also suppose that $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$. Then, $\lim_{x\to a} f(x) = L$. This is known as the:

   A. Bounded Value Theorem B. Central Limit Theorem
   C. Intermediate Value Theorem D. Mean Value Theorem E. NOTA

4. Use the theorem presented in question 3 to evaluate

$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$$

   A. -1 B. 0 C. 1 D. does not exist E. NOTA

Consider the following information for questions 5 through 7.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	12	5	7	4
2	2	15	3	10
3	1	6	11	3
4	9	14	8	16

Assume that $f : [1, 4] \to \mathbb{R}$ and that $g : [1, 4] \to \mathbb{R}$ are both differentiable.

5. Let $s(x) = f(x) - g(x)$. Find $s'(3)$.

   A. -10 B. -5 C. 0 D. 3 E. NOTA

6. Let $p(x) = f(x)g(x)$. Find $p'(4)$.

   A. 72 B. 254 C. 256 D. 296 E. NOTA

7. Let $r(x) = \frac{f(x)}{g(x)}$. Find $r'(1)$.

   A. $-\frac{64}{49}$ B. $-\frac{13}{49}$ C. $\frac{13}{49}$ D. $\frac{5}{4}$ E. NOTA

8. Find the minimum distance between the origin and the set $\{(x, y)|y - \sqrt{x+2} = 0\}$.

A. $\frac{\sqrt{7}}{2}$ B. $\sqrt{2}$ C. $\frac{7}{4}$ D. 2 E. NOTA

9. The probability of spending t minutes waiting in line for a particular ride is $p(t)$ where

$$p(t) = \begin{cases} 0 & t < 0 \\ \frac{t}{400} & 0 \le t \le 20 \\ \frac{400}{t^3} & t > 20 \end{cases}$$

Find the mean waiting time, in minutes, assuming that you can wait for an infinite amount of time.

A. 10 B. 20 C. $\frac{80}{3}$ D. 40 E. NOTA

10. Eli is selling frisbees for \$10 each. Given that he buys them at a cost of $\$(0.2x^2 - 10x + 5)$ for x frisbees, how many frisbees should Eli buy (and then sell for \$10) in order to maximize his profit?

A. 0 B. 20 C. 50 D. 500 E. NOTA

11. The Frankenator didn't realize that curvature would be on the Calculus BC test. He was horrified when he read "29. Find the value of x at the point of greatest curvature on the curve $y = \ln x$." Unfortunately, he couldn't remember that curvature can be found as $\kappa = \frac{y''(x)}{\left(1 + (y'(x))^2\right)^{\frac{3}{2}}}$, so he ripped apart his test in frustration. What was the correct answer to this question?

A. $\frac{\sqrt{2}}{2}$ B. $\frac{1+\sqrt{3}}{2}$ C. 1 D. $\frac{1}{2}$ E. NOTA

12. Given that $f'(x) = |x|$, is $f(x)$ continuous on the domain $[-1, 1]$? Is $f(x)$ differentiable on $[-1, 1]$?

A. yes, yes B. yes, no C. no, yes D. no, no E. NOTA

13. Hannah is riding a dinosaur with position function $s(t) = (\sin t + \cos t)^2$. At what speed is she traveling when $t = \frac{\pi}{6}$?

A. $\frac{1}{2}$ B. $\frac{\sqrt{3}}{2}$ C. 1 D. $\sqrt{3}$ E. NOTA

14. Given that f is a monic polynomial of degree 3 and $f(2) = f'(2) = f''(2) = 0$, find $f(3)$.
Note: A monic polynomial is a polynomial with a leading coefficient of 1.

A. -1 B. 0 C. 1 D. 27 E. NOTA

15. The area bounded by the parabola $y(x)$ and the $x-$axis, where $y(r_1) = y(r_2) = 0$ and $\{r_1, r_2\} \in \mathbb{R}$, can be given generally as $k \cdot |y(0)| \cdot |r_1 - r_2|$ for some constant k. Find the value of k.

A. $\frac{1}{2}$ B. $\frac{5}{8}$ C. $\frac{2}{3}$ D. $\frac{3}{4}$ E. NOTA

16. Find $p + q + r + s + t$ given that p, q, r, s, and t are rational numbers such that

$$\int_0^{\pi} x^4 \sin(x)\, dx = p + q\pi + r\pi^2 + s\pi^3 + t\pi^4$$

A. 0 B. 11 C. 13 D. 37 E. NOTA

17. Evaluate $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x$.

A. 1 B. $\sqrt{e}$ C. e^2 D. ∞ E. NOTA

18. Consider the following pairs in the form $(f(x), x_0)$. When attempting to use Newton's Method to estimate a root of $f(x)$ using the initial guess x_0, one of these pairs will not allow you complete the first iteration (that is, to find x_1) due to division by 0. Find that pair.

A. $(x^2 - 1,\ 1)$ B. $(2x^2 + 32,\ 4)$ C. $(x,\ 0)$ D. $(x^2 + 2x,\ -1)$ E. NOTA

19. Evaluate

$$\int_0^{\frac{5}{6}} \frac{dx}{\sqrt{25 - 9x^2}}$$

A. $\frac{\ln 3}{10}$ B. $\frac{\pi}{18}$ C. $\frac{5}{18}$ D. $\frac{\pi}{9}$ E. NOTA

20. Let $f(g(x)) = \frac{d}{dx}(g(x))$. Find $\frac{d}{dx}(f(2010x^2 + 1))$.

A. 4020 B. $4020x$ C. $2010x^2$ D. $2010x^2 + 1$ E. NOTA

21. Given that $[[x]]$ represents the greatest integer function, evaluate

$$\lim_{x\to 4} x[[x]]$$

A. 4 B. 12 C. 16 D. does not exist E. NOTA

22. When the length of the space diagonal of a cube is x, the volume of a cube is increasing at a rate $\sqrt{3}$ times greater than the rate at which the surface area of the cube is increasing. Give the value of x.

A. 4 B. $2\sqrt[4]{27}$ C. $4\sqrt{3}$ D. 12 E. NOTA

23. Find the value of $\frac{1}{2} - \frac{1}{2^3 \cdot 3} + \frac{1}{2^5 \cdot 5} - \frac{1}{2^7 \cdot 7} + \frac{1}{2^9 \cdot 9} - \dots$

A. $\ln(\frac{1}{2})$ B. $\cos(\frac{1}{2})$ C. $\sin(\frac{1}{2})$ D. $e^{\frac{1}{2}}$ E. NOTA

24. Given that $g(x) = \begin{cases} xg(x-1) & x > 0 \\ 1 & x = 0 \end{cases}$ and $f(a,b) = \dfrac{a^b}{g(b)}$, find $\ln\left[\displaystyle\prod_{i=1}^{n}\left(\sum_{j=0}^{\infty} f(i,j)\right)\right]$.

A. 1 B. n C. $\frac{n(n+1)}{2}$ D. $n!$ E. NOTA

25. Find the number of critical points of the function $f(x) = \dfrac{x-2}{\sqrt{x}}$.

A. 0 B. 1 C. 2 D. 3 E. NOTA

26. Given that $p(x)$ defines a probability density function, find k.

$$p(x) = \begin{cases} ke^x & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

A. 0 B. $\frac{1}{e}$ C. $\frac{1}{e-1}$ D. 1 E. NOTA

27. Consider that Waldo follows the path in the xy-plane defined by the parametric equations

$$x(t) = e^t + 2 \quad \text{and} \quad y(t) = \cos t$$

At $t = 0$, where's Waldo?

A. $(2,0)$ B. $(2,1)$ C. $(3,0)$ D. $(3,1)$ E. NOTA

28. Using the information given in the previous question, at what positive value of t is Waldo traveling with the greatest velocity?

A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. $\frac{5\pi}{4}$ D. $\frac{3\pi}{2}$ E. NOTA

29. The function $y(x)$, with domain $(0,\pi)$, satisfies $\sin(x)\dfrac{dy}{dx} + y\cos(x) = \sqrt{x}$.

Given that $y\left(\dfrac{\pi}{3}\right) = 0$, find $y\left(\dfrac{2\pi}{3}\right)$.

A. $\dfrac{4\pi^{3/2}}{3\sqrt{3}}\left(2^{3/2}-1\right)$ B. $\dfrac{4\pi^{3/2}}{9}\left(2^{3/2}-1\right)$ C. $\dfrac{4\pi^{3/2}}{9\sqrt{3}}\left(2^{3/2}-1\right)$ D. $\dfrac{4\pi^{3/2}}{27}\left(2^{3/2}-1\right)$ E. NOTA

30. Find the interval of convergence of the folowing series:

$$1 + \frac{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots}{1!} + \frac{(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots)^2}{2!} + \frac{(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots)^3}{3!} + \dots$$

A. $[1,-1]$ B. $(-1,1]$ C. $(-\infty,\infty)$ D. $(-\infty,-1)\cup(1,\infty)$ E. NOTA