QUESTION 1

Let

$$\begin{aligned} A &= \text{The value of } x^2, \text{ where } (x)^2 + (x+1)^2 + (x-1)^2 = 365. \\ B &= \text{The value of } x^2 + \frac{1}{x^2}, \text{ given that } x + \frac{1}{x} = 9. \\ C &= \text{The sum of the roots in the equation } 2 + \sqrt{3x - 8} = x. \\ D &= \text{The value of } 2x - y \text{ given that} \end{aligned}$$

$$\frac{2y - 3x}{3} = \frac{11}{3} \text{ and } \frac{-7y + 2x}{3} = -\frac{13}{3}$$

Compute the value of $A + B + C + D$.

QUESTION 2

The solution to the inequality $|4x - 12| < 8$ can be written in interval notation in the form (x_1, x_2). Let A equal the sum $x_1 + x_2$.

The solution to the inequality $x^2 + 5x + 6 < 0$ can be written in interval notation in the form (x_3, x_4). Let B equal the sum $x_3 + x_4$.

Let C equal the constant term of $\frac{f(x-3)}{14}$ where $f(x-1) = x^2 + 6x - 2$ and let D equal $f(7)$ for this same function.

Compute the value of $A + B + 7C + D$.

QUESTION 3

Let

$$\begin{aligned} A &= x, \text{ where } \log_2(\log_3 x) = 1. \\ B &= (\log_2 3)(\log_9 2) \\ C &= \log_4 16 + \log_3 \frac{1}{3} \\ D &= \text{The value of } y \text{ when the expression } \frac{1}{\log_6 x} + \frac{1}{\log_7 x} \text{ is written in the form } \log_x y. \end{aligned}$$

Compute the value of $3A + 2B + C + D$.

QUESTION 4

Let $\Psi(f)$ be the function that gives a value of 0 if f is an odd function, 1 if f is an even function, and 2 if f is neither an odd nor even function. Let

$$\begin{aligned} A &= \Psi(y = -3x^2 + 4). \\ B &= \Psi(y = 2x^3 - 5x^2 + 2x + 4). \\ C &= \Psi(y = x^3 + x). \\ D &= \Psi(y = x). \end{aligned}$$

Compute $A + B + C + D$.

QUESTION 5

Let

$$\begin{aligned} A &= \text{The value of } f(5) \text{ if } f \text{ is a cubic polynomial with roots } 1, -2, 6 \text{ and } f(0) = -36. \\ B &= \text{The value of } g(f(2)) \text{ where } f(x) = x^2 - 1 \text{ and } g(x) = 2x. \\ C &= \text{The number of positive roots of the equation } x^3 + 5x^2 + 2x - 9 = 0. \\ D &= \text{The sum } xy + yz + xz \text{ where } x, y, z \text{ are the roots to the cubic } x^3 - 7x^2 + 12x = 0. \end{aligned}$$

Compute the value of $A + B + C + D$.

QUESTION 6

For

$$f(x) = \begin{cases} f(x-2) + f(x-3) & x \geq 6 \\ \dfrac{(x+2)!}{(x-1)!} - 5 & 1 \leq x < 6 \\ \dfrac{\sqrt{24} - \sqrt{|x|}}{\sqrt{12} - \sqrt{3}} & x < 1 \end{cases},$$

let

$$\begin{aligned} A &= f(8) \\ B &= f(6) \\ C &= f(1) \\ D &= f(-6) \end{aligned}$$

Compute the value of $A + B + C + D$.

QUESTION 7

Let

$$\begin{aligned} A &= \sum_{n=0}^{13}(n+1). \\ B &= 1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}\cdots. \\ C &= \prod_{n=1}^{3}(n^3-5). \end{aligned}$$

I drop a ball off the top of an 80 foot building. If the ball rebounds to $\frac{3}{8}$ its height everytime it hits the ground, let D equal the total distance travelled by the ball.

Compute the value of $A+B+C+D$.

QUESTION 8

Consider the conic section

$$4x^2 + 9y^2 - 24x + 36y + 5 = -31.$$

Let

$$\begin{aligned} A &= \text{The area of the conic section.} \\ B &= \text{1 if the conic section is a hyperbola, 2 if it is an ellipse, or 3 if it is a circle.} \\ C &= \text{the length of the minor axis.} \\ D &= \text{the length of the major axis.} \end{aligned}$$

Compute the value of $ABCD$.

QUESTION 9

Let

$$\begin{aligned} A &= \text{The sum of the coefficients in the expansion of } (3x^2+2y-7z)^5. \\ B &= \text{The constant term of the expansion of } \left(4x^2+\frac{1}{x}\right)^6. \\ C &= \text{The coefficient of the } x^3y \text{ term in the expansion of } (x+y)^4. \end{aligned}$$

Compute the value of $-\dfrac{A}{8}+\dfrac{B}{15}+C$.

QUESTION 10

Pamela is attempting to solve this systems of equations using Cramer's Rule:

$$\begin{aligned} 3x + 5y + z &= 10 \\ 2x - y + 7z &= 4 \\ -x - 4y - 6z &= -7 \end{aligned}$$

Each variable x, y, and z can be solved by dividing its corresponding matrix $[G]_x, [G]_y$, and $[G]_z$, respectively, by a matrix $[H]$.

Let

$$\begin{aligned} A &= \text{The determinant of the matrix } [G]_x \text{ used to solve for } x. \\ B &= \text{The determinant of the matrix } [G]_y \text{ used to solve for } y. \\ C &= \text{The determinant of the matrix } [G]_z \text{ used to solve for } z. \\ D &= \text{The determinant of the matrix } [H] \text{ used to solve for } x, y, \text{ and } z. \end{aligned}$$

Compute the value of $A + B + C - D$.

QUESTION 11

The Sachdeva Construction Company has to rebuild Rickards High School. It sends 3 men to rebuild the school. Suppose the each of the 3 men can rebuild one wing of the school in 9 hours. Assume for all parts that all men work at constant rates. Consider the following situations:

- Let A equal the time required in hours to complete one wing of the school if all 3 men work on it together.
- Suppose all 3 men initially begin working on the wing of the school together. After one hour, one man leaves, and the remaining two men complete the wing together. Let B equal the total amount of time in hours it takes to complete the wing.
- Suppose the three men bring three other men who work at the same rate (each can build one wing of the school in 9 hours). Let C be equal to the number of hours it would take to complete the wing if all 6 men work on it together.
- The three men work continuously together for 21 hours. The number of wings they complete is equal to D.

Compute the value of $A + B + C + D$.

QUESTION 12

Let

$$\begin{aligned} A &= x, \text{ where } 4x(\ln \sqrt[7]{e^{21}})^x = 72. \\ B &= \text{The sum of the solutions to the equation } e^{x^3-5x^2-4x+2} = 1. \\ C &= \text{The value of } e \text{ rounded to the nearest tenth.} \end{aligned}$$

The expression $e^{2\ln 3 - \ln 4x + \ln x^2}$ can be simplified into the form $\dfrac{ax}{b}$, where a and b are relatively prime positive integers. Let $D = a + b$.

Compute the value of $A + B + \frac{C}{3} + D$.

QUESTION 13

Let A equal the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

A pyramid of pencils has 11 pencils in the top row, 12 pencils in the next row, 13 in the following row, etc. If there are 19 rows of pencils, let B equal the total number of pencils.

A culture of bacteria grows exponentially, doubling every hours. If there are 17 bacteria at hour 0, then C is the first hour that begins with at least 1000 bacteria.

Compute the value of $A + B + C$.

QUESTION 14

Let

A = The number of distinct ways to place 6 distinct beads on a bracelet with a clasp.

B = The number of distinct ways to place 6 distinct beads on a bracelet without a clasp.

C = The number of distinct three person committees that can be formed from a group of 9 students, where 2 certain students refuse to be on the committee together.

$D = ({}_5C_4)({}_5P_4)$.

Compute the value of $A + B + C + D$.