Question 1. A rectangle has side lengths x and y. Find the following values in terms of x and y.

A. Find the area of the rectangle.
B. Find the perimeter of the rectangle.
C. Find the length of the diagonal of the rectangle.
D. Find the value of x in terms of y that maximizes the area of the rectangle if $x + y$ is constant.

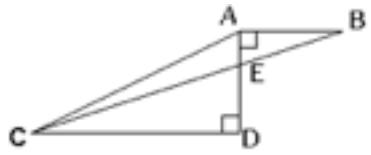
Question 2. Determine if each statement is "always true", "sometimes true", or "never true":

A. Two lines that do not intersect are parallel.
B. The sum of the angles of a triangle is 180 degrees.
C. A square is a rhombus.
D. A right triangle is an obtuse triangle.

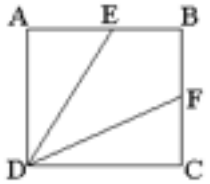
Question 3. Let $S(A)$ be the number of sides of shape A, and $M(A)$ be the measure of one interior angle of regular polygon A. For example, when A is a square, $S(A) = 4$ and $M(A) = 90°$.

A. A regular polygon P has 1337 sides. What is $S(P)$?
B. Let H be a regular hexagon. Find $S(H)$.
C. Let H be a regular hexagon. Find $M(H)$.
D. Let X be a regular dodecagon. Find $M(H)$.

Question 4.

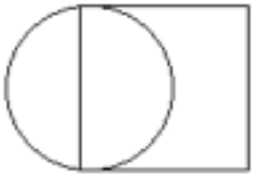
A. 

$AD = 4, AB = 3$, and $CD = 9$. What is the area of triangle AEC ?

B. 

$ABCD$ is a square of side 3, and E and F are the midpoints of sides AB and BC respectively. What is the area of the quadrilateral $EBFD$?

C. All the 9 dots in a 3 by 3 array (grid) are 2 units apart vertically and horizontally. What is the length of the longest line segment that can be drawn joining any two points in the array without passing through any other point?

D. 
In the figure above the square has two sides which are tangent to the circle. If the area of the circle is $4a^2\pi$, what is the area of the square in terms of a?

Question 5.

A. Evaluate $\binom{7}{3}$. That is, the number of ways to choose 3 people from a group of 7.
B. Find the number of diagonals in a regular hexagon.
C. Find the sum of the interior angles, in degrees, of a regular heptagon (also called a septagon).
D. Find the sum of the exterior angles of a regular hexadecagon.

Question 6. Find the area of each triangle, given some of its properties.

A. Sides of length 6, 9, and 11
B. Perimeter of 22 and inradius 5
C. Sides of length 69, 92, and 115
D. Base of 10 and height 20

Question 7.

A. Find the value, in degrees, of the largest angle of a triangle in which the ratio of the angles of the triangle are $3 : 5 : 10$.
B. Find the number of square inches in the area of a right triangle with hypotenuse of length 18 and legs with lengths in the ratio $2 : 1$.
C. Find the number of faces of a polyhedron with 10 vertices and 6 edges.
D. A triangle has a perimeter 13. The two shorter sides have integer lengths equal to x and $x + 1$. Find the sum of the possible lengths of the longest side.

Question 8. Determine if each statement is "always true", "sometimes true", or "never true".

A. Three unique points determine a plane.
B. The angle bisectors of a triangle intersect at the circumcenter.
C. The volume of a regular triangular pyramid (tetrahedron) with edge length e is $\frac{e^3\sqrt{2}}{12}$.
D. The area of a circle with radius r is larger than its diameter.

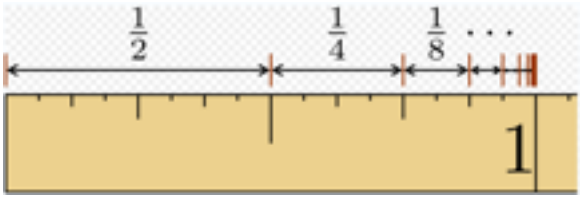
Question 9. This diagram shows a circle inscribed in an equilateral triangle with side length 2:

A. What is the area of the triangle?
B. What is the radius of the circle?
C. What is the area of the region inside the triangle but outside the circle?
D. Let D_n be the number of diagonals in a regular $n-$gon. What is D_5?

Question 10. Consider a circle with radius 3.

A. What is the area of the circle?
B. What is the length of a 72 degree arc of this circle?
C. If the circle were rotated about its diameter, what would the volume of the resulting 3d solid be?
D. What is the area of the largest triangle that can be inscribed in this circle?

Question 11. Here's a hint for the **geometric** sequences (parts A and B):



A. Evaluate $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$
B. Evaluate $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\cdots$
C. Find $\frac{1}{n}+\frac{1}{n^2}+\frac{1}{n^3}+\frac{1}{n^4}+\cdots$ in terms of n
D. Evaluate $(x-a)(x-b)(x-c)(x-d)\cdots(x-z)$

Question 12. The area of a triangle with two sides of length a and b that intersect at an angle of θ is

$$\frac{1}{2}ab\sin\theta$$

A. Find the area of a triangle with $a=5$, $b=4$, and $\theta=90°$.
B. Find the area of a triangle with $a=4$, $b=4$, and $\theta=60°$.
C. Find θ when a triangle with $a=2$ and $b=2$ has area $\sqrt{3}$.
D. Find b when a triangle with $a=1$ and $\theta=45°$ has area 5.