1. For each of the following statements answer "True" or "False".

   (a) There exists a figure of infinite surface area but finite volume.

   (b) Every function

   $$f : \mathbb{R} \to \mathbb{R}$$

   is continuous somewhere.

   (c) Every continuous function is differentiable.

   (d) For any continuous function G

   $$\int \frac{dG(x)}{dx} dx = G(x) + c$$

   where c is a constant.

2. Which of the following converge to a finite number. For each part your answer should be either "Converges" or "Does Not Converge" appropriately.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{-n}}$ (b) $\displaystyle\int_1^{\infty} \frac{1}{x^2}$ (c) $\displaystyle\sum_{n=1}^{\infty} \left|\frac{\sin(n)}{n}\right|$ (d) $\displaystyle\sum_{k=1}^{\infty} (-1)^k/k$

3. Evaluate the following symbolically, produce an expression valid over the respective domain of definition: (a) $\displaystyle\frac{dx}{\sqrt{a^2 - x^2}}$ (b) $\displaystyle\int \frac{dx}{(x^2+1)^2}$ (c) $\displaystyle\int \arctan(x) dx$ (d) $\displaystyle\int \ln(x) dx$

4. Take the derivative of the following expressions with respect to x:

   (a) e^2 (b) 2^e (c) $3x^2 + \sin(x) + 1337$ (d) $\arctan(x) + x$

5. Evaluate:

(a) $\lim_{x\to 3}\left(\frac{x^3-3x^2-x+3}{x^2-5x+6}\right)$ (b) $\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos^2(x)-\sin^2(x)}{2\sin(x)\cos(x)}\right)$

(c) $\lim_{x\to\infty}\left(\sqrt[3]{x^3+5x^2}-x\right)$ (d) $\lim_{x\to 0}\left(\frac{\sin(x+y)-\sin(y)}{x}\right)$

6. For the polynomial $f(x)=x^3-14x^2+49x-36$ find each of the following: Note: $x=1$ is a root of $f(x)$.

(a) Find the region R_1 in $\mathbb{R}$ where f is increasing.

(b) Find the region R_2 in $\mathbb{R}$ where f is concave up.

(c) Find the region R_3 in $\mathbb{R}$ where f is increasing, concave up and satisifies $3y\leq 14$ for $y\in R_3$.

(d) Find the region R_4 in $\mathbb{R}$ where f has an inflection point.

7. Find:

(a) The area under the curve $y=x^2+3x+1$ and above the x-axis bounded by $x=3$ and $x=5$.

(b) The volume of rotation of the region bounded by $y=4-x^2$ and $y=0$ about the y-axis.

(c) The area of the region bounded by $y=\frac{1}{x^2}$, $y=0$ and $x=1$.

(d) The volume of revolution of the region bounded by $y=x^3$, $x=2$ and $y=0$ about the x-axis.

8. Evaluate:

(a) $\frac{d}{dx}\int_{-x^3}^{x^2}\sin(t)dt$ (b) $\int\frac{d(e^{x+t})}{dt}dx$ (c) $\frac{d}{dx}\int_{-x}^{x}e^{x+t}dt$ (d) $\int_{x-y}^{x+y}\frac{d(\sin(t))}{dt}dt$

9. Do
   (a) Approximate $e^{.01}$ via the tangent line to e^x at $x = 0$.
   (b) Approximate $f(x) = \frac{1}{\sqrt{x^3+1}}$ via the tangent line at $x = 2$.
   (c) Find the slope of the tangent line to $f(x) = \arcsin(\log(\sqrt{x}))$ at $x = e$.
   (d) Find the equation of the normal line of $y = \sin(x)$ at $x = \frac{\pi}{2}$.

10. Do
    (a) A five meter ladder is leaning against a vertical wall. The base slips away at the constant rate of $\frac{1}{2}m/s$. How fast is the top of the ladder moving down when the bottom of the ladder is $3m$ away from the wall in m/s?
    (b) The radius of a circle decreases at $1/2m/s$. What is the rate of change of area when the radius is $4m$?
    (c) If ice cream is being poured into an inverted circular cone at a rate of $3in^3/sec$ and the cone has base radius $4in$ and height $8in$ find the rate at which the ice cream is rising when it is filled $3in$ high, from the bottom of the cone.
    (d) If a spherical fake pumpkin is filling so that the volume increases at a rate of $50cm^3/sec$. What is the diameter of the pumpkin when the radius is expanding at a rate of $12.5cm/sec$?

11. Evaluate:
    (a) $\int (\sin(x) + \cos(x))^2 dx$
    (b) Find the maximum value of $y = \sin(x)\cos(x)$ on the interval $[0, \frac{\pi}{2}]$.
    (c) Find the arc length of $-\ln(|\cos(x)|)$ on the interval $[\frac{\pi}{6}, \frac{\pi}{3}]$.
    (d) Simplify the following expression: $\tan(2\sin^{-1}(\frac{1}{3}))$.

12. Solve:
    (a) How many real roots does $x^3 - 4x^2 - 11x + 30$ have?
    (b) What is the average speed of Bob who travels 50m in 10 seconds, in m/s?
    (c) Find the inverse of the function $y = \frac{e^x - e^{-x}}{2}$.
    (d) Simplify: $e^{i\theta} + e^{-i\theta}$