Calculus Individual Solutions

1. (D) For the function to be continuous at x=0, the left-hand limit, right-hand limit, and value of f at x=c must all be equal. The left-hand limit is $|0+3|^2 = 9$ and the right-hand limit is $0^2 + 9 = 9$. Thus, f(0)=c=9.

2. (B) Substituting x=9 gives the indeterminate form 0/0.

Solution A: $\lim_{x\to 9}\frac{3-\sqrt{x}}{x-9} = \lim_{x\to 9}\left[\left(\frac{3-\sqrt{x}}{x-9}\right)\left(\frac{3+\sqrt{x}}{3+\sqrt{x}}\right)\right] = \lim_{x\to 9}\frac{-1}{3+\sqrt{x}} = \frac{-1}{6}$

Solution B: By L'Hopital's, $\lim_{x\to 9}\frac{3-\sqrt{x}}{x-9} = \lim_{x\to 9}\left(\frac{-\frac{1}{2\sqrt{x}}}{1}\right) = \lim_{x\to 9}\frac{-1}{2\sqrt{x}} = \frac{-1}{6}$

3. (A) The distance is $\frac{|1\cdot 3 + 2\cdot 2 + 3\cdot 1 - 4|}{\sqrt{3^2+2^2+1^2}} = \frac{6}{\sqrt{14}} = \frac{3\sqrt{14}}{7}$.

4. (D) The derivative gives no obvious solutions. However, it is clear that as x gets larger, the $3x^2$ term is much more important to maximize rather than miniscule changes in the fluctuating sinx term. Thus, we want the value of x at the end of the interval, 2π. So, $f(2\pi) = 1 + \sin(2\pi) + 3(2\pi)^2 = 1 + 12\pi^2$.

5. (D) $\int_{-\infty}^{\infty}\frac{1}{1+x^2}dx = \arctan x\Big|_{-\infty}^{\infty} = \arctan\infty - \arctan(-\infty) = \frac{\pi}{2} - \left(\frac{-\pi}{2}\right) = \pi$. The positive integers less than pi are 1, 2 and 3, for a total of 3.

6. (B) $f'(x) = 2x - 1$. $x_1 = 2 - \frac{f(2)}{f'(2)} = \frac{4}{3}$. $x_2 = \frac{4}{3} - \frac{f(4/3)}{f'(4/3)} = \frac{16}{15}$.

7. (A) The graphs intersect at x=0 and x=1. $\int_0^1 (x^2 - x)dx = \frac{1}{3}x^3 - \frac{1}{2}x^2\Big|_0^1 = \frac{1}{6}$.

8. (D) $f'(c) = \frac{f(4)-f(0)}{4-0} = 5$. $f'(c) = 1 + 2c$. $1 + 2c = 5 \rightarrow c = 2$.

9. (C) $ydy = e^x dx$. $\int ydy = \int e^x dx$. $\frac{1}{2}y^2 = e^x + c$. Substituting the given value (0,1), we find that c=-1/2. So, $y = \pm\sqrt{2e^x - 1}$. However, referring back to the original equation, the negative solution is clearly extraneous. So, $y(1) = \sqrt{2e - 1}$.

10. (C) Using partial fractions:
$$\int_2^5 \frac{4x^2-2}{x^3-x}dx = \int_2^5 \left(\frac{2}{x}+\frac{1}{x+1}+\frac{1}{x-1}\right)dx = \left(2\ln x+\ln(x+1)+\ln(x-1)\right)\Big|_2^5 = \ln 50.$$

11. (A) The acceleration is the derivative of the velocity, and the velocity is the derivative of the position/height. Since $a(t)=-32$, $v(t)=-32t+c$, and since the initial velocity (t=0) is 40, c=40. Thus, $h(t)=-16t^2+40t+C$, but since it is shot from the ground, the initial height (t=0) is 0, so C=0. The maximum value of h occurs where t=5/4, and $h(5/4)=25$.

12. (A) $\lim_{x\to 0}\frac{\sin(x)(1-\cos x)}{x^2} = \lim_{x\to 0}\frac{\sin(x)}{x}\cdot\lim_{x\to 0}\frac{1-\cos x}{x} = 1\cdot 0 = 0.$

13. (B) $f^{-1}(25) \Rightarrow f(X)=25 \Rightarrow X=3.$

14. (B) The rate of change, or derivative, is cosx. On this interval, the cosine ranges from [-1,0]. Thus, the maximum is 0.

15. (D)
$$\int_0^1 (\sin x+\cos x+\frac{1}{x+1}+3x)dx = (-\cos x+\sin x+\ln(x+1)+\frac{3}{2}x^2)\Big|_0^1 = \frac{2\ln 2+2\sin 1-2\cos 1+5}{2}.$$

16. (A) The derivative of a constant is 0.

17. (D) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (2-2\sin x)dx = (2x+2\cos x)\Big|_{\frac{-\pi}{2}}^{\frac{\pi}{2}} = 2\pi.$

18. (E) $\lim_{x\to\infty}\frac{x^5+6x^3+8x^2+5x+1}{x^4\sin x+7x^3+9x^5+3} = \lim_{x\to\infty}\frac{x^5}{9x^5} = \frac{1}{9}.$

19. (B) Integration by parts: $\int x\ln x dx = \frac{1}{2}x^2\ln x - \int\frac{1}{2}xdx.$

$$\left(\frac{1}{2}x^2\ln x-\frac{1}{4}x^2\right)\Big|_1^e = \frac{e^2+1}{4}.$$

20. (C) Integration by parts: $\int\sin^{-1}xdx = x\sin^{-1}x-\int\frac{x}{\sqrt{1-x^2}}dx.$ → C

21. (C) Quotient rule: $h'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{g^2(x)} = \frac{2\cdot 3-1\cdot 4}{9} = \frac{2}{9}.$

22. (C) $2\pi\int_0^{\pi} x^3 \sin x dx = 2\pi^2(\pi^2 - 6)$.

23. (C) Let $x = \tan\theta$. Then, $dx = \sec^2\theta$. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sec^2\theta}{\sec^4\theta} d\theta = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta d\theta = \frac{\pi}{2}$.

24. (D) By definition.

25. (C) $f'(x) = 3x^2 + 6x + 4$. The sum of the roots is -6/3 = -2.

26. (C) By definition, $x^2 + y^2 = r^2 = 3$.

27. (B) If x is negative, we have a fraction minus an integer, which cannot equal an integer. If x is 0, we obtain 1, not 2. If x=1, we have a solution. If x is greater than 1, the first term is much larger than the second. The only solution is x=1.

28. (A) The derivative of a constant is 0.

29. (A) $\int(6x^2 - 2x + 2)x = 2x^3 - x^2 + 2x$. The sum of the squares of the roots is equal to the [sum]^2-2[sum taken 2 at a time] = $\left(\frac{1}{2}\right)^2 - 2(1) = \frac{-7}{4}$.

30. (D) T=30+(90-30)e^{kt}. The given information tells us that:

60=30+(60)$e^{.5k}$, so k=2ln(1/2)=ln(1/4). Thus, the temperature after another half hour is 30+60$e^{\ln(1/4)}$=30+15=45 F.