

Question 1. Consider the circle with equation $(x - 5)^2 + (y - 7)^2 = 9$.

- A. Find the radius of the circle.
- B. Find the shortest distance from the center of the circle to the line $15x - y - 7 = 0$.
- C. The circle intersects the line $x = 8$ at the point (a, b) . What is $a + b$?
- D. Find the shortest distance from the circle to the line $15x - y - 7 = 0$.

Question 2. Evaluate the following, where $i = \sqrt{-1}$ and \bar{z} denotes the complex conjugate of z :

- A. $\left| \frac{1}{3-4i} \right|$
- B. $|\bar{z}| - \bar{z}$, where $z = 3 + 5i$
- C. $9i - 16i^2 + 25i^3 - 36i^4$
- D. $(i + 1)^{800} - (i - 1)^{800}$

Question 3.

- A. Solve for x : $\log_4(x - 3) + \log_4(x + 3) = 2$.
- B. Solve for x : $\sqrt{2x + 7} - x = 2$.
- C. Fully simplify: $\frac{\ln 4 + \ln 3 - \ln 2}{2 \ln 3 + 2 \ln 2}$.
- D. Find a if $x = \log_3 3$ and $xa = \log_5 25$.

Question 4. Sums and products of roots.

- A. Find the sum of the distinct roots of the equation $x^2 - 6x + 9 = 0$.
- B. Find the sum of the integral roots of the equation $7x^3 - 5x^2 - 7x + 5 = 0$.
- C. Find the sum of the roots of $x^5 - 5x^3 + 3x^2 - x + 2 = 0$.
- D. Find the product of the roots of $x^4 - 7x^2 + 8x^2 + 2x + 9 = 1$.

Question 5. Solve for x in the following equations.

- A. $x \log(x \log(x \log \dots = 1000$ B. $|x + 1| = 5$ C. $\sqrt{2x - 3} = x$ D. $\log 3 + \log 3 = \log x$

Question 6.

- A. In how many zeroes does $50!$ end? (How many zeroes follow the last non-zero digit?)
B. Find the third term in the binomial expansion of $(2x + 3)^8$.
C. Simplify: $\left(\frac{512}{27}\right)^{\frac{2}{3}}$
D. Find the sum of the roots of the equation $|x - 9| = 4x + 3$.

Question 7. Let $f(x) = x + 3$, $g(x) = \frac{x+2}{5}$, and $h(x) = 3x + 1$. Note that f^{-1} denotes the inverse of f .

- A. Evaluate $g(8)$. B. Evaluate $f^{-1}(4) \cdot g^{-1}(4) \cdot h^{-1}(4)$.
C. Find the remainder when $x^4 - 6x^3 - 9x^2 + 20x + 7$ is divided by $f(x)$. D. Evaluate $f(1)$.

Question 8.

- A. Find the maximum value of $-x^2 - 6x + 16$. B. Find the minimum value of $|3x - 6| + 2$.
C. Solve for x : $|x^2 + x| = 12$. D. Find the determinant of the inverse of $\begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 3 \\ 4 & 5 & 6 \end{pmatrix}$.

Question 9.

- A. How many 3 letter strings can be formed if the first letter must be a vowel (only English letters)?
 B. How many numbers are in the sequence 1, 5, 9, 13, ..., 2005, 2009?
 C. What is the simplified value of $\sqrt{17 - 12\sqrt{2}}$?
 D. Find $a^3 + b^3$ for real numbers a and b such that $a + b = 5$ and $ab = 3$.

Question 10.

- A. z is inversely proportional to the square of y and directly proportional to the square root of x . Find the constant of proportionality in terms of x , y , and z .
 B. Given that $a * b = \left(\frac{ab+b^2+4}{a-b}\right) \left(\frac{a^2+b^2+a+7}{b}\right)$, find $4 * 2$.
 C. Given that $p = \log 7$ and $q = \log 3$, find $\log 147$ in terms of p and q .
 D. If x satisfies the equation $(\log_2 x)^2 + 2\log_2 x - \log_2 8 = 0$, what is 4^x ?

Question 11.

- A. What is the eccentricity of a circle?
 B. Find the eccentricity of the ellipse $\frac{(x-3)^2}{49} + \frac{(y-1)^2}{25} = 1$.
 C. Find the area of the region bounded by the graph of $3y^2 = 9 - x^2$.
 D. Find the length of the latus rectum of the ellipse $\frac{(x-1)^2}{7} + \frac{(y-1)^2}{13} = 1$.

Question 12. Consider

$$f(x) = \begin{cases} -x + 3 & x < 0 \\ 50x + 1 & 0 \leq x < 3 \\ f(x-2) + f(x-1) & x \geq 3 \end{cases}$$

- A. $f(-1)$ B. $f(2)$ C. $f(3)$ D. $f(6)$