QUESTION 1

Let:

$$\begin{aligned} A &= \text{The exact value of } \sin(75^\circ) \\ B &= \text{The exact value of } \cos(165^\circ) \\ C &= \text{The exact value of } \tan\left(\frac{3\pi}{8}\right) \\ D &= \text{The exact value of } \tan\left(\frac{11\pi}{8}\right) \end{aligned}$$

Find $A + B + C + D$.

QUESTION 2

Let:

$$\begin{aligned} A &= \sin(x)\cos^2(x)\tan^2(x)\csc^4(x)\cot^3(x)\sec^2(x)\sin^2(x)\tan(x) \\ B &= (\csc(x)-\cot(x))(\csc(x)+\cot(x)) \\ C &= \frac{\sqrt{2-2\cos(2x)}}{2} \\ D &= \sin(x)\sin(3x)+\cos(x)\cos(3x) \end{aligned}$$

Find $ABCD$, in simplest form and in terms of x, assuming $0 < x < \frac{\pi}{2}$.

QUESTION 3

Given the polar equation:

$$r = 2017\cos(2017\theta)$$

Let:

$$\begin{aligned} A &= \text{The number of petals in the polar graph of this curve} \\ B &= \text{The length of one petal, as measured from the origin to the furthest tip} \\ C &= \text{The number of times the polar graph of this equation intersects the polar graph of } r = 420 \\ D &= \text{The distance from the origin to the point where } \theta = \frac{\pi}{2017} \end{aligned}$$

Find $\dfrac{A}{C} + \dfrac{B}{D}$.

QUESTION 4

Let:

$$\begin{aligned} A &= \cos(45) + i\sin(45) \\ B &= \cos(75) + i\sin(75) \\ C &= \cos(76) - i\sin(76) \\ D &= \sin(0) + i\cos(0) \end{aligned}$$

Find the argument of $ABCD$, for $i = \sqrt{-1}$. Note: All angles are in radians.

QUESTION 5

Find:

$$\begin{vmatrix} 1 & 3 & 2 & 1 \\ 2 & 1 & -1 & 1 \\ 3 & 0 & 4 & 1 \\ 3 & 2 & 3 & 1 \end{vmatrix}$$

QUESTION 6

For parts C & D, choose the most specific name. Let:

$$A = \text{The eccentricity of the conic given by } \frac{2017}{8+16\sin(\theta)}$$

$$B = \text{The length of the latus rectum of } x^2+9y^2-4x-72y+139=0$$

C = The number of letters in the name of the conic described by the parametric equations

$$x = 3\sec(t)+1$$
$$y = 6\tan(t)+2$$

D = The number of letters in the name of the conic described by the parametric equations

$$x = \sin(t)+6$$
$$y = \cos(t)-9$$

Find $A+B+C+D$.

QUESTION 7

Given:

$$f(x) = x^5 + 6x^4 - 4x^3 + 106x^2 - 5x - 680;\ f(\sqrt{5}) = 0 \text{ and } f(-8) = 0$$

Let:

$$\begin{aligned} A &= \text{The number of complex roots} \\ B &= \text{The sum of the real roots} \\ C &= \text{The product of the nonreal roots} \\ D &= \text{The sum of the squares of the real roots} \end{aligned}$$

Find $A + B + C + D$.

QUESTION 8

Let:

$$A = \text{The ratio of } a \text{ to } b \text{ if } a \text{ and } b \text{ are positive and } \frac{a}{b} = \frac{a+b}{a}$$

$$B = \text{The length of } AB\text{, given that } ABC \text{ is an isosceles triangle with base angles } B \text{ and } C \text{ equal to } 36^o \text{ and point } D \text{ on } BC \text{ such that } AD = CD = 2 \text{ (Hint: } \sin 54^o = \frac{\sqrt{5}+1}{4}\text{)}$$

Find $\frac{A}{B}$.

QUESTION 9

Let:

$$
\begin{aligned}
A &= \text{The distance from the point } (3,5) \text{ to the line } y = -\frac{3}{4}x + 3 \\
B &= \text{The distance from the point } (12,16) \text{ in rectangular coordinates to the point } (15, \arctan\left(\frac{4}{3}\right) + \frac{\pi}{2}) \\
&\quad \text{in polar coordinates} \\
C &= \text{The shortest distance from the graph } r = \pi \text{ to the graph } -\frac{209}{r} = r - 30\sin\theta \\
D &= \text{The } x\text{-value of the polar coordinate } (2\pi, \frac{\pi}{6}) \text{ in rectangular coordinates}
\end{aligned}
$$

Find $A + B + C + D$.

QUESTION 10

Let:

$$\begin{aligned} A &= \ln(a), \text{ where } a \text{ is the first quadrant root of } x^2 = i \\ B &= \ln(b), \text{ where } b \text{ is the root of } x^5 = 1 \text{ with argument within } (0, \frac{\pi}{2}) \\ C &= \ln(c), \text{ where } c \text{ is the root of } x^8 = 1 \text{ with the 3rd smallest argument} \\ D &= \text{The value of } \frac{e^{i\theta} - e^{-i\theta}}{2i}, \text{ expressed in terms of } \theta \end{aligned}$$

Find $A + B + C + D$, for $i = \sqrt{-1}$.

QUESTION 11

Beginning with an initial value of 0, add the value of each true statement and subtract the value of each false statement below to find the final answer.

- (4) In a cyclic quadrilateral, the sine of opposite angles are always equal.
- (6) In a cyclic quadrilateral, the cosine of opposite angles sum to 0.
- (-5) A singular matrix has a determinant of 1.
- (12) If U, V, and W are three-dimensional vectors and $\times$ denotes the cross product, $U\times(V+W) = (U\times V)+(U\times W)$.
- (1) A continuous function is either always increasing or always decreasing.
- (-14) The amplitude of the graph of $y = 3\sqrt{7}\sin(7x) - 6\sqrt{7}\cos(7x)$ is $3\sqrt{14}$.
- (-10) A Gaussian Integer is a complex number $z = a + bi$ such that a and b are both integers.
- (2017) The probability that $f(x) = 4$ given the function $f(x) = x^2$ when x is chosen from the interval $[-4, 4]$ is $\frac{2}{9}$.

QUESTION 12

Let:

$$\begin{aligned} A &= \text{The vector resulting from } <3,6,2> + <2,6,1> \\ B &= \text{The vector resulting from } <3,6,2> - <2,6,1> \\ C &= \text{The vector resulting from } <3,6,2> \times <2,6,1> \\ D &= \text{The value of } <3,6,2> \bullet <2,6,1> \end{aligned}$$

Find the sum of the components of $A + DB + C$.

QUESTION 13

Joshua has two urns. In one urn there are eight green marbles, three red marbles, and four blue marbles. In a second urn there are five yellow marbles, six red marbles, and nine blue marbles. Joshua randomly selects an urn and randomly draws a marble from it. If Joshua drew a red marble, what is the probability that the urn he drew from was the first urn?

QUESTION 14

Let:

$$
\begin{aligned}
A &= \text{The distance traveled by an ant walking on } r = \cos(\theta) \text{ for } \theta = [0, 2\pi] \\
B &= \text{The value of } a \text{ in the equation } -3e^{3-a} + 2e^{-2a+6} = 20 \\
C &= \text{The value of } WAT \text{ in the decomposition of } \frac{4x^2 + 13x - 7}{x^3 + 6x^2 - x - 30} \text{ as expressed in the form } \frac{W}{x+i} + \frac{A}{x+j} + \frac{T}{x+k} \\
D &= \text{The value of } |my| \text{ given } m = \sqrt{-2i},\ y = |m|, \text{ and } i = \sqrt{-1}
\end{aligned}
$$

Find $A + B + C + D$.