

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

- $\sec(\operatorname{arccot}(-4)) =$

(A)  $\sqrt{17}$                       (B)  $-\sqrt{17}$                       (C)  $-\frac{\sqrt{17}}{4}$                       (D)  $\frac{\sqrt{17}}{4}$                       (E) NOTA
- What is the period of the function  $y = 4 \sin\left(\frac{2x}{3}\right) + 5 \cos\left(\frac{x}{2}\right) - 2 \tan(x)$ ?

(A)  $4\pi$                       (B)  $6\pi$                       (C)  $12\pi$                       (D)  $\frac{\pi}{3}$                       (E) NOTA
- Shreya the cheese maker cuts a cheese block into the shape of a parallelepiped determined by the vectors  $\langle 1, 0, 3 \rangle$ ,  $\langle 2, 4, 2 \rangle$ , and  $\langle -1, 3, 4 \rangle$ . What is the volume of the cheese block in units<sup>3</sup>?

(A) 20                      (B) 40                      (C) 48                      (D) 64                      (E) NOTA
- Nihar is trying to hit his PR (personal record) on the bench press. The weight he can lift over time can be modeled by a sinusoidal wave. The maximum weight he can lift on a given day is 135 lbs. while his minimum weight is 75 lbs. Given that shortest time he must wait before he can lift his maximum again is 2 weeks, what function below models the weight he can bench over time? (Note.  $y =$  weight, in pounds;  $x =$  time in weeks)

(A)  $y = 135 \sin(\pi x) - 75$                       (B)  $y = 75 \sin(2\pi x) + 60$   
 (C)  $y = 60 \sin(2x) + 75$                       (D)  $y = 60 \cos(2x) + 75$                       (E) NOTA
- Given the summation below:

$$a + bi = \sum_{k=0}^{194} e^{\frac{(2k+1)\pi i}{195}}$$

Find the value of  $b$ .

(A) 0                      (B) 1                      (C)  $e$                       (D) -1                      (E) NOTA
- What is the graph of the polar function  $r = 4 \cos(\theta)$  on the interval of  $0 \leq \theta \leq 2\pi$ ?

(A) Line                      (B) Circle                      (C) Limaçon                      (D) Rose                      (E) NOTA
- Cherry ate garlic bread. Her parents, Sidhika and Qing, notice the rancid smell and start running away from her. Using their handy compasses, Sidhika runs northwest at 4 m/s away from Cherry, and Qing runs  $75^\circ$  north of the east line at 3 m/s. How far are they from each other after 10 seconds in meters?

(A)  $\sqrt{13}$                       (B)  $\sqrt{17}$                       (C) 5                      (D) 50                      (E) NOTA
- Let  $x = \arccos\left(\frac{3}{5}\right) + \arcsin\left(\frac{8}{17}\right)$ . What is value of  $\tan(x)$ ?

(A)  $\frac{32}{45}$                       (B)  $\frac{84}{45}$                       (C)  $\frac{42}{13}$                       (D)  $\frac{84}{13}$                       (E) NOTA
- Let  $S$  be the sum of the infinite geometric sequence:  $16, 16 - 8\sqrt{3}, 28 - 16\sqrt{3}, \dots$  Find the value of  $4S$ .

(A)  $\frac{16\sqrt{3}}{2}$                       (B) 32                      (C)  $64 + 64\sqrt{3}$                       (D)  $\frac{32\sqrt{3}}{3}$                       (E) NOTA
- Simplify  $i^{2017}$ .

(A)  $e^{\frac{\pi}{2}}$                       (B)  $e^\pi$                       (C)  $e^{-\frac{\pi}{2}}$                       (D)  $e^{\frac{\pi i}{2}}$                       (E) NOTA

11. For  $x \neq \frac{\pi}{2}$ ,

$$3 \sin^3(x) \cdot \left( \sum_{n=0}^{\infty} \cos^{2n}(x) \right) - 4 \sin(x) \cdot \left( \sum_{n=0}^{\infty} ((-1)^n \cot^{2n}(x)) \right) =$$

- (A)  $\sin^3(x)$       (B)  $\cos^3(x)$       (C)  $\sin(3x)$       (D)  $\cos(3x)$       (E) NOTA

12. Let matrix  $I = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 0 \\ 0 & 3 & -1 \end{bmatrix}$ . Find the value of  $I_{(3,3)}^{-1}$ , the value of the element in the third row and column in the inverse matrix of  $I$ .

- (A) -1      (B) 0      (C) 3      (D)  $\frac{5}{7}$       (E) NOTA

13. Jessica, Cassie, and Sriraj are playing a dice game. They each take turns rolling the dice until one person wins.

- If Jessica rolls a 2 or a 3, then she wins.
- If Cassie rolls a 1 or a 4, then she wins.
- If Sriraj rolls a 5 or a 6, then he wins.

Jessica starts the game and Cassie goes after her, each rolling their dice once. Sriraj goes last but unlike those two, he rolls his dice twice at each of his turns. If all three of them went and no one won, the cycle starts over. What is the probability that Sriraj wins?

- (A)  $\frac{2}{3}$       (B)  $\frac{4}{27}$       (C)  $\frac{4}{13}$       (D)  $\frac{2}{13}$       (E) NOTA

14. What is the distance between the polar points  $(9, \frac{\pi}{3})$ , and  $(4, \frac{4\pi}{3})$  ?

- (A) 5      (B) 13      (C)  $\sqrt{133}$       (D)  $\sqrt{62}$       (E) NOTA

15. Which of the following vectors is orthogonal to  $\langle -4, 1, -3 \rangle$ ?

- (A)  $\langle 2, 3, 1 \rangle$       (B)  $\langle -3, 0, -4 \rangle$       (C)  $\langle -2, -2, 2 \rangle$       (D)  $\langle 3, 0, 4 \rangle$       (E) NOTA

16. Sohan's jawline can be modeled by the equation  $y = -2x + 4$  from  $0 \leq x \leq 2$ , when he is looking straight ahead. When Sohan looks up, the equation modeling his jawline is rotated  $45^\circ$  counterclockwise about the y-intercept. Given that  $(0, 4)$  and  $(2, 0)$  are the starting and ending points of his chin, respectively, find the distance traveled by the endpoint of his chin when he goes from looking straight ahead to looking up.

- (A)  $\frac{\pi}{4}$       (B)  $\frac{\pi\sqrt{5}}{4}$       (C)  $\frac{\pi\sqrt{5}}{2}$       (D)  $\pi\sqrt{5}$       (E) NOTA

17. The  $\lim_{x \rightarrow 3} f(x)$  given that  $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{at } x \neq 3 \\ 4 & \text{at } x = 3 \end{cases}$

- (A) 0      (B) 4      (C) 6      (D) DNE      (E) NOTA

18. What type of conic is the following function:  $2x^2 - 6xy + 3y^2 + 4x + 2y + 3000 = 0$

- (A) Parabola      (B) Circle      (C) Ellipse      (D) Hyperbola      (E) NOTA

19. Find the product of the eigenvalues of the matrix:  $\begin{bmatrix} 3 & 0 & 2 \\ 1 & 3 & 2 \\ 2 & 3 & 4 \end{bmatrix}$

- (A) 16      (B) 8      (C) 4      (D) 0      (E) NOTA

20.  $\arccos(\sin(\sin(\cot(\arcsin(\tan(\frac{\pi}{4})))))) =$   
 (A)  $\frac{\pi}{2}$  (B)  $-\frac{\pi}{2}$  (C) 0 (D)  $\pi$  (E) NOTA

21. As a result of the “Basel Problem”, we have the identity:

$$\sum_{k=0}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$$

Let  $C = \{1, 5, 7, 11, 13, \dots\}$  or the the set of all values not divisible by 2 or 3. Let:

$$\sum_{c \in C} \frac{1}{c^2} = \frac{a}{b} \cdot \pi^2$$

where  $a$  and  $b$  are relatively prime integers. Find  $a + b$ .

- (A) 10 (B) 15 (C) 36 (D) 37 (E) NOTA
22. Let  $f(x) = \sqrt{\frac{\cos^2(x) - \sin^2(x)}{1 - \tan^2(x)}}$ . Evaluate  $f(\frac{3\pi}{8})$ .  
 (A)  $-\frac{\sqrt{2 - \sqrt{2}}}{2}$  (B)  $\frac{\sqrt{2 - \sqrt{2}}}{2}$  (C)  $-\frac{\sqrt{2 + \sqrt{2}}}{2}$  (D)  $\frac{\sqrt{2 + \sqrt{2}}}{2}$  (E) NOTA

23. If  $\vec{a} = j + 2k$ ,  $\vec{b} = 7i + 8j + 9k$ ,  $\vec{c} = 12k + 4j + 3i$ ,  $\vec{d} = 13k + 6j + 5i$ ,  $\vec{e} = a \times b$  and  $\vec{f} = c \times d$ , then solve for the angle between  $\vec{e}$  and  $\vec{f}$ .

- (A)  $\sin(\frac{32\sqrt{30}}{195})$  (B)  $\cos(\frac{32\sqrt{30}}{195})$  (C)  $\cos^{-1}(\frac{32\sqrt{30}}{195})$  (D)  $\sin^{-1}(\frac{32\sqrt{30}}{195})$  (E) NOTA

24. What is the product of the phase shift and the vertical shift of the periodic function  $y = \sin(13x - 1)$ ?

- (A)  $\frac{5}{13}$  (B) 0 (C)  $\frac{10\pi}{13}$  (D)  $\frac{2\pi}{169}$  (E) NOTA

25. Given the equations  $x = 4 - 2t$  and  $y = 3 + 6t - 4t^2$ , what is the bound of  $y$  on the interval  $0 \leq t \leq 3$ ?

- (A)  $(-\infty, \frac{21}{4})$  (B)  $(-\infty, \frac{21}{4}]$  (C)  $[-15, \frac{21}{4}]$  (D)  $(-15, \frac{21}{4})$  (E) NOTA

26. Calculate the volume of the solid of revolution formed by rotating the finite region bounded by the graphs of  $f(x) = \sin(x) + 2$  and  $f(x) = 1$  from the interval  $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$ ,  $240^\circ$  about the x-axis.

- (A)  $3\pi$  (B)  $9\pi^2 - 2\pi$  (C)  $7\pi$  (D)  $3\pi^2$  (E) NOTA

27. Homer Simpson loves integration. Although he was previously named “Homer Run”, he changed his last name to Simpson to show his devotion toward his favorite form of integral approximation, Simpson’s rule . Using Simpson’s rule with  $n = 26$ , Homer correctly approximates the value of

$$\int_{-3}^5 (x^3 + 3x^2) dx$$

What does Homer obtain?

- (A) 37.5 (B) 108.625 (C) 144 (D) 288 (E) NOTA

28. Evaluate:

$$\frac{d}{dx} \int_0^{x^2} (t^3 + 3t^2) dt$$

- (A) 0 (B)  $x^3 + 3x^2$  (C)  $2x^7 + 6x^5$  (D)  $x^6 + 3x^4$  (E) NOTA

29. Let  $f(x)$  be a cubic polynomial with  $f(0) = 1$ ,  $f(1) = 3$ ,  $f(2) = 15$ ,  $f(3) = 43$ . Find  $f(4)$ .

(A) 40

(B) 57

(C) 63

(D) 93

(E) NOTA

30. What  $x$ -value satisfies the Mean Value Theorem for Derivatives for the function  $f(x) = x^2 + 3x + 5$ , on the interval  $(1, 3)$ ?

(A)  $\frac{3}{2}$

(B) 2

(C)  $\frac{5}{2}$

(D) There is no point

(E) NOTA