For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Find the area of triangle ABC, given that $\angle A = 105°$, $\angle B = 15°$, and side $AC = 5$.
   (A) $\frac{75+50\sqrt{3}}{4}$ (B) $\frac{25(\sqrt{6}+\sqrt{2})}{2}$ (C) $\frac{15\sqrt{3}}{2}$ (D) $\frac{25\sqrt{3}}{4}$ (E) NOTA

2. Which of the following statement(s) is/are always true:

   I. $\tan^2(\theta) + 1 = \sec^2(\theta)$ II. $\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$

   III. $\sin^2(\theta) + \cos^2(\theta) = 1$ IV. $\cos(2\theta) = 2\sin^2(\theta) - 1$

   (A) I, II, IV (B) I, III (C) I, II, III (D) I, II, III, IV (E) NOTA

3. What is the midpoint of the focus of $y = \frac{1}{3}x^2 + 6x + 19$, and the center of $4x^2 + 9y^2 + 24x - 36y - 92 = 0$?
   (A) $\left(-3, \frac{49}{8}\right)$ (B) $\left(0, \frac{49}{8}\right)$ (C) $\left(-3, \frac{33}{8}\right)$ (D) $\left(-3, \frac{47}{8}\right)$ (E) NOTA

4. Let $f(x) = \log_4 x$, $g(x) = \log_x 64$, and $h(x) = \sqrt{x-10}$. What is the domain of $h((fg)(x))$?
   (A) $(-\infty, \infty)$ (B) $(0, \infty)$ (C) $[10, \infty)$ (D) $\varnothing$ (E) NOTA

5. If $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$, and $\sin(\theta) = \frac{8}{17}$, then what is $\cos(\theta)$?
   (A) $-\frac{15}{17}$ (B) $\frac{15}{17}$ (C) $-\frac{8}{17}$ (D) $\frac{8}{15}$ (E) NOTA

6. Jasmine is traveling on the vector $\langle 6, 3, -2\rangle$ and Kyle is traveling on the vector $\langle 5, 4, -1\rangle$. Given that θ is equal to the angle between the two vectors, find $\sin(\theta)$.
   (A) $\frac{\sqrt{1029}}{61}$ (B) $\frac{7\sqrt{1281}}{61}$ (C) $\frac{44\sqrt{61}}{61}$ (D) $\frac{21\sqrt{237}}{61}$ (E) NOTA

7. Which of the following polar equations is a hyperbola with an eccentricity of $\frac{3}{2}$ and a directrix of $x = 4$?
   (A) $r = \frac{24}{2+3\cos(\theta)}$ (B) $r = \frac{2}{3+6\cos(\theta)}$ (C) $r = \frac{2}{3+6\sin(\theta)}$ (D) $r = \frac{24}{1+3\cos(\theta)}$ (E) NOTA

8. Aditya is standing on a street corner, and wants to cross the road. He looks directly in front of him and sees a stop sign, 5 feet away. Then he turns his head 75° to the right, and spots Meit eating ice cream 12 feet away. What is the distance between Meit and the stop sign?
   (A) $\sqrt{169+30\sqrt{6}+30\sqrt{2}}$ (B) $\sqrt{169+30\sqrt{6}-30\sqrt{2}}$
   (C) $\sqrt{169-30\sqrt{6}+30\sqrt{2}}$ (D) $\sqrt{169-30\sqrt{6}-30\sqrt{2}}$ (E) NOTA

9. Let A be a 3×3 matrix with a determinant of 3. What is the determinant of $3A$?
   (A) 3 (B) 9 (C) 27 (D) 81 (E) NOTA

10. Find the $\lim_{x\to 3} \frac{x^3+6x^2-7x-60}{x^3+x^2-24x+36}$.
    (A) $\frac{64}{13}$ (B) $\frac{13}{4}$ (C) $\frac{56}{9}$ (D) $\frac{44}{7}$ (E) NOTA

11. What is the amplitude of $f(x) = 5\cos(\theta) + 12\sin(\theta)$?
    (A) 6 (B) 13 (C) 17 (D) 24 (E) NOTA

12. Given that $\vec{a}$ is $\langle 1, -2, 7\rangle$ and $\vec{b}$ is $\langle 3, 5, -4\rangle$, find the projection of $\vec{a}$ on $\vec{b}$.
(A) $\left\langle -\frac{20}{27}, \frac{40}{27}, \frac{140}{27}\right\rangle$ (B) $\left\langle -\frac{14}{27}, \frac{140}{27}, \frac{77}{27}\right\rangle$ (C) $\left\langle -\frac{34}{27}, \frac{32}{27}, \frac{20}{27}\right\rangle$ (D) $\left\langle -\frac{26}{27}, \frac{7}{27}, \frac{155}{27}\right\rangle$ (E) NOTA

13. Wenxin likes chickens. She wants to make a parallelepiped pen for her chickens bounded by the vectors $\langle 6, 3, -2\rangle$, $\langle 2, 3, -4\rangle$, and $\langle 5, -3, 1\rangle$. What is the volume of her pen?
(A) 42 (B) 78 (C) 104 (D) 133 (E) NOTA

14. Which of the following rectangular coordinates is equivalent to the polar coordinate $(6, 60)$?
(A) $(3\sqrt{2}, 3\sqrt{2})$ (B) $(3, 3\sqrt{3})$ (C) $(\sqrt{3}, 3)$ (D) $(-\sqrt{3}, 3\sqrt{3})$ (E) NOTA

15. Evaluate $\displaystyle\sum_{n=1}^{6} 3\left|e^{\frac{(n+1)(\pi)i}{3}} - e^{\frac{(n-1)(\pi)i}{3}}\right|$
(A) 18 (B) $18\sqrt{3}$ (C) $6\sqrt{3}$ (D) 6 (E) NOTA

16. The roots of the equation $x^6 = 729$ are plotted in the Argand plane. Find the area bounded by these points.
(A) $\frac{9\sqrt{3}}{2}$ (B) $\frac{27\sqrt{3}}{4}$ (C) $\frac{27\sqrt{3}}{2}$ (D) $27\sqrt{3}$ (E) NOTA

17. Let $\arccos\left(\frac{3}{5}\right) + \arccos\left(\frac{8}{17}\right) = \arctan\left(\frac{a}{b}\right)$, where $a > 0$ and $b < 0$. Find $a + b$ if all the angles are within the first quadrant.
(A) 41 (B) 40 (C) 39 (D) 38 (E) NOTA

18. What is the period of $f(x) = \sin(3x) + \cos(2x)$?
(A) π (B) 3π (C) 6π (D) 2π (E) NOTA

19. Find the distance between the polar points $(6, 75°)$ and $(3, -15°)$.
(A) $3\sqrt{3}$ (B) $3\sqrt{5}$ (C) 3 (D) 6 (E) NOTA

20. Evaluate: $\det\left(\begin{bmatrix}\operatorname{cis}(15°) & \operatorname{cis}(30°)\\ \operatorname{cis}(60°) & \operatorname{cis}(15°)\end{bmatrix}^{2016}\right)$
(A) -1 (B) 0 (C) 1 (D) 2 (E) NOTA

21. Find the positive difference between the solutions of $\log_2(\log_4 x^2) = \log_8(\log_2 x^8)$
(A) $2^{2\sqrt{2}}$ (B) $2^{2\sqrt{2}-1}$ (C) $2^{\sqrt{6}}$ (D) $2^{2\sqrt{2}+1}$ (E) NOTA

22. Simplify $\dfrac{(1-\sin^2(x))(1-\cos^2(x))}{\cos(x)\sin(x)\tan(x)(1+\sin(x))}$.
(A) $\cos(x)$ (B) $\sin(x)$ (C) $1-\cos(x)$ (D) $1-\sin(x)$ (E) NOTA

23. Evaluate the following infinite geometric series: $(16+8i) + (12-4i) + (4-8i) + \ldots$
(A) $7+15i$ (B) $5+3i$ (C) $16+8i$ (D) $24-8i$ (E) NOTA

24. $f(x) = 3x+1$. Let $f_2(x) = f(f(x))$, $f_3(x) = f(f(f(x)))$, etc. Find $f_5(1)$.
(A) 363 (B) 364 (C) 365 (D) 724 (E) NOTA

25. $|z-(2i)||z-(2i)| = 32$ is graphed in the Argand plane. What type of conic is this? (Choose the most specific option.)
(A) Parabola (B) Circle (C) Ellipse (D) Hyperbola (E) NOTA

26. Haley likes to run. If her position function is modeled by $s(t) = \sec(t) + \tan(t) + \sin(t)$, what is her acceleration at $t = \frac{\pi}{4}$?

(A) $\frac{3\sqrt{2}}{2} + 2$ (B) $\frac{3\sqrt{2}}{2} + 4$ (C) $\frac{5\sqrt{2}}{2} + 2$ (D) $\frac{5\sqrt{2}}{2} + 4$ (E) NOTA

27. Evaluate: $\int_0^{\frac{\pi}{3}} \sec^3(x)\, dx$. (Hint: Use the identity $\sec^2(\theta) = \tan^2(\theta) + 1$)

(A) $\frac{2\sqrt{3} + \ln(2+\sqrt{3})}{2}$ (B) $\frac{\frac{3}{2} + \ln\left(\frac{3\sqrt{3}}{2}\right)}{2}$ (C) $\frac{2 + \sqrt{3} + \ln(2\sqrt{3})}{2}$ (D) $\frac{2\sqrt{6} + \ln(2\sqrt{6})}{4}$ (E) NOTA

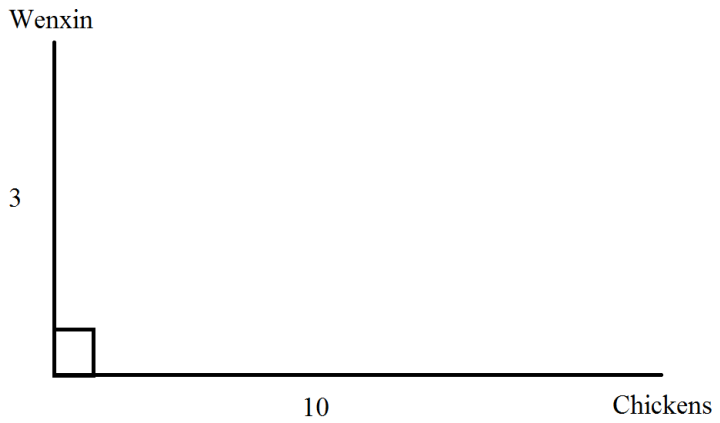
28. Kim has a spherical Tupperware with negligible thickness that has volume of 288π. What is the volume of the largest cone that can be placed inside?

(A) 4π (B) $12\sqrt{3}\pi$ (C) $\frac{256\pi}{3}$ (D) $\frac{288\pi}{3}$ (E) NOTA

29. Carson is memorizing factorials, but he forgot three digits of 21!. On his sheet of paper, he wrote: $21! = 51090abc171709440000$. What is the value of $c + 100b + 10a$, if a, b, and c are the correct missing digits?

(A) 492 (B) 420 (C) 402 (D) 942 (E) NOTA

30. Oh No! Wenxin is stranded from her chickens. She is on a boat 3 miles from the coast, and her chickens are on the coast, 10 miles east of her location. She can row at 2 mph, and can run at 6 mph. What distance should she run on land, to get to her chickens the fastest? Keep in mind that she can't row on land, nor run underwater. All the answers are given in miles, and the diagram is not drawn to scale.



(A) $\frac{9\sqrt{2}}{4}$ (B) $10 - \frac{9\sqrt{2}}{4}$ (C) $10 - \frac{3\sqrt{2}}{4}$ (D) $\frac{3\sqrt{2}}{4}$ (E) NOTA