

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Assume that all inverse trigonometric functions have their traditional restricted ranges. Let $i = \sqrt{-1}$. Good Luck!

- Find the cross product of the vectors $\langle 1, 8, 2 \rangle$ and $\langle 2, 1, 4 \rangle$.
 (A) $30i + 8j - 15k$ (B) $30i - 15k$ (C) $34i - 15k$ (D) $30i + 15k$ (E) NOTA
- How many solutions exist to the following system of equations?

$$b = a^3 + 2a$$

$$c = b^3 + 2b$$

$$a = c^3 + 2c$$
 (A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA
- The number 12A3B4 is divisible by 11 with tens digit B and thousands digit A. How many possible combinations of (A,B) are there?
 (A) 8 (B) 7 (C) 9 (D) 6 (E) NOTA
- Let $\log(\cos \theta) = A$ and $\log(\sin \theta) = B$. Express $\log \frac{\sin(2\theta)}{2}$ in terms of A and B.
 (A) AB (B) $2B$ (C) $A + B$ (D) B (E) NOTA
- What is the area bounded by the function $f(x) = \sqrt{4 - x^2}$ in the first quadrant?
 (A) $\frac{\pi}{2}$ (B) π (C) 2π (D) 4π (E) NOTA
- What is the distance between the point $(1, 0, -2)$ and the plane $x + 2y + z = 4$?
 (A) 1 (B) 6 (C) $\frac{5\sqrt{6}}{6}$ (D) $\frac{5}{6}$ (E) NOTA
- The force exerted by the magnetic field on a positively charged particle is defined by the cross product of the particles velocity and the magnetic field. What is the magnitude of the force exerted on a positively charged particle with velocity vector $\langle 5, 5, 0 \rangle$ by the magnetic field defined by $\langle 0, 0, -5 \rangle$?
 (A) 25 (B) $25\sqrt{2}$ (C) $25\sqrt{3}$ (D) 50 (E) NOTA
- Siddarth is standing near the base of a 30 meters tall building and looks up at a 45° angle and sees Awnish at the top. Awnish jumps and falls for 10 seconds before opening a parachute. At this instant, Siddarth has to look up at a 30° to see Awnish. How far in meters did Awnish fall before opening his parachute?
 (A) $30 - 10\sqrt{3}$ (B) $30 + 10\sqrt{3}$ (C) $10\sqrt{3}$ (D) 10 (E) NOTA
- What is the slope of the tangent line to the function $f(x) = \sqrt{9 - x^2}$ at the point $(0, 3)$?
 (A) 0 (B) -1 (C) 1 (D) $\frac{1}{2}$ (E) NOTA
- What is the equation of the tangent line to the function $f(x) = \sqrt{9 - x^2}$ at the point $(0, 3)$?
 (A) $x + y = 3$ (B) $y = 9$ (C) $y = 3$ (D) $x - y = 3$ (E) NOTA
- In polar, what is the graph of $r = 2 + 2\sin(\theta)$?
 (A) rose (B) circle (C) lemniscate (D) limacon (E) NOTA

12. A fly is at the point (1,2,3) and flies in a straight line at a constant speed. If the fly reaches the point (2,4,-1) after 1 second, where will the fly be after 6 seconds?
 (A) (7,14,-21) (B) (7,14,21) (C) (6,12,-17) (D) (7,-14,-21) (E) NOTA
13. What is the volume of a solid created by the rotation of the function $f(x) = 4$ about the x -axis given the restriction $2 < x < 6$?
 (A) 16π (B) 32π (C) 64π (D) 128π (E) NOTA
14. Given $f(x) = x^2 - 2x + 1$, what is $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$?
 (A) 0 (B) 1 (C) 2 (D) 4 (E) NOTA
15. On the Argand plane what is the distance between the complex numbers $3+2i$ and $4-11i$?
 (A) $\sqrt{170}$ (B) 13 (C) $\sqrt{155}$ (D) 14 (E) NOTA
16. The equation $4x^2 + 6xy - 3y^2 + 5x - 7y = 15$ represents what type of figure?
 (A) circle (B) parabola (C) hyperbola (D) ellipse (E) NOTA
17. What is $\cos(3\theta)$?
 (A) $4\cos^3(\theta) - 3\cos(\theta)$ (B) $3\cos^3(\theta) - 4\cos(\theta)$ (C) $3\sin(\theta) - 4\sin^3(\theta)$ (D) $4\sin^3(\theta) - 3\cos(\theta)$ (E) NOTA
18. Given vector $v = \langle 1, 3 \rangle$ and vector $u = \langle 3, 1 \rangle$, find $|\text{proj}_v u|$.
 (A) 6 (B) $\frac{6}{\sqrt{13}}$ (C) $\frac{3\sqrt{10}}{5}$ (D) $\langle 0, 0, -8 \rangle$ (E) NOTA
19. We know that the rate of change of the volume of a sphere at any given moment is the surface area of the sphere at that moment, times the rate of change of the radius at that moment. Using this, what is the rate of change of the radius of a sphere, in cm per minute, at the moment when the radius is 5 cm given that the volume is increasing at a constant rate of 1 cm^3 per minute?
 (A) $\frac{1}{100\pi}$ (B) $\frac{1}{50\pi}$ (C) $\frac{1}{25\pi}$ (D) $\frac{1}{4\pi}$ (E) NOTA
20. What is $\left| \text{cis}^6\left(\frac{\pi}{12}\right) \right|$?
 (A) $\frac{\sqrt{2}}{2}$ (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$ (E) NOTA
21. 70 percent of people who have raideritis will have a positive result when tested. 85 percent of people who do not have raideritis will have a negative result when tested. If 12 percent of people have raideritis, what is the probability of randomly selecting a person, and getting someone who tests negative?
 (A) 0.784 (B) 0.366 (C) 0.3606 (D) 0.36006 (E) NOTA
22. What is the slope of the tangent line to the function $f(\theta) = \text{cis}(\theta)$ at $\theta = \frac{\pi}{4}$ in the Argand Plane?
 (A) -1 (B) 0 (C) 1 (D) $\frac{\sqrt{2}}{2}$ (E) NOTA
23. What is the period of the graph of $f(x) = 6\cos^2(5x) - 6\sin^2(5x)$?
 (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{5}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{10}$ (E) NOTA

24. If we have a secant line of a function with a slope of 0, then we know that somewhere between the intersection points of the secant line and the function, there exists a point at which the tangent line to the function is 0. If a secant line intersects the function $f(x) = x^2 - 1$ at the points $(-1, 0)$ and $(1, 0)$, what point on the function $f(x)$ has a tangent line with slope equal to the slope of this secant line?
- (A) $(1, 0)$ (B) $(-1, 0)$ (C) $(0, -1)$ (D) $(0, 1)$ (E) NOTA
25. If $x = \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{2}{3}\right)$, find $\tan x$.
- (A) $\frac{3}{2}$ (B) $\frac{7}{4}$ (C) 2 (D) $\frac{5}{2}$ (E) NOTA
26. Rithik is standing, looking up at the top of the statue of the great Kethireddy. Rithik is 4 feet tall and the statue is 10 feet tall. If the sine of the angle that Rithik must look up to look at the top of the statue is equal to $\frac{\sqrt{3}}{3}$, what is the distance from Rithik's head to the top of the statue?
- (A) $6\sqrt{3}$ (B) $6\sqrt{2}$ (C) 6 (D) 12 (E) NOTA
27. Given that $a + b + c = 1$, find the maximum value of $ab + ac + bc$.
- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) NOTA
28. What is the surface area of the solid of revolution of the function $f(x) = x$ about the y -axis with the restriction $-3 < x < 2$?
- (A) 13π (B) $13\pi\sqrt{2}$ (C) $13\pi + 9\pi\sqrt{2}$ (D) $13\pi(1 + \sqrt{2})$ (E) NOTA
29. What is the average value of the function $f(x) = x^2 + 1$ on the interval $[0, 2]$?
- (A) 2 (B) 4 (C) $\frac{7}{3}$ (D) $\frac{14}{3}$ (E) NOTA
30. Let $S = \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^9} + \frac{1}{2^{16}} + \frac{1}{2^{25}} + \dots$. Find $[1000S]$ where $[x]$ represents the greatest integer function.
- (A) 562 (B) 563 (C) 564 (D) 565 (E) NOTA