QUESTION 1

Given

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	0	7	0	32
2	3	2	$\frac{1}{3}$	-3
7	3	6	-3	$\frac{1}{3}$

Let

$$\begin{aligned} A &= \frac{d}{dx}\left[f(g(x))\right]\Big|_{x=1} \\ B &= \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]\Big|_{x=7} \\ C &= \frac{d}{dx}f(x) + f'(x) - g'(x)\Big|_{x=2} \\ D &= \int_7^1 f'(x) - g'(x)\,dx + \left(\frac{d}{dx}\left[f(x) - g(x)\right]\Big|_{x=2}\right) \end{aligned}$$

Find $A - \dfrac{19C}{BD}$.

QUESTION 2

Let

$$
\begin{aligned}
A &= \lim_{x\to\infty} \frac{7x^2+8}{9x^2-12} \\
B &= \lim_{x\to 0} \frac{\arctan(6x)}{\sin(24x)} \\
C &= \lim_{x\to\frac{\pi}{12}} \ln\sqrt{\sec(2x)} \\
D &= \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} \text{ if } y=\sqrt{x}\cot(x)
\end{aligned}
$$

Find $A+B+e^{2C}+D^2$.

QUESTION 3

Let

$$\begin{aligned} A &= \int_1^5 3x^2 + 5x^4 \, dx \\ B &= \int_{-\infty}^{-1} \frac{-1}{x^4} \, dx \\ C &= \frac{d}{dx} \int_0^{4x} \left(\csc\theta - \sec\theta\right) \, d\theta \\ D &= \int_0^{\frac{\pi}{4}} \sec^3\theta \, d\theta \end{aligned}$$

Find $\dfrac{A+3B}{3247} + C\Big|_{x=\frac{\pi}{24}} - 2D$.

QUESTION 4

Of course we all know that position, velocity, and acceleration are related by differentiation and integration, but few know that there are 4 more classifications of change in motion. The derivative of acceleration is jerk, the derivative of jerk is snap, the derivative of snap is crackle, and the derivative of crackle is pop.

Given a particle's position is given by the function $p(t) = t^6 - t^5 + 4t^3 + 7t^2 - t + 9$ where t is time, let

A = The jerk at time $t = 0$.

B = The snap at time $t = 5$.

C = The pop at time $t = 2$.

D = The closest integer to the value of the distance traveled by the particle between $t = 0$ and $t = 1$ minus the displacement of the particle.

Find $ABCD$.

QUESTION 5

Let

$$A = \int_0^{2\pi} |\sin(x)|\,dx$$

$$B = \int_0^{2\pi} x\cos(x)\,dx$$

$$C = \int_0^{2\pi} e^{2x}\cos(3x)\,dx$$

Find $A + B + C$.

QUESTION 6

The region bounded by $y = x^3$ and $y = x$ is called E. Let

A = The area of region E.

B = The volume of the solid generated by rotating region E about the x-axis.

C = The volume of the solid generated by rotating region E about the y-axis.

D = The value of b such that $\int_0^b x^2\,dx$ is equal to the volume of the solid generated by rotating region E about the x-axis.

Find $A + \frac{7B}{5} + CD\left(\frac{\pi}{7}\right)^{\frac{-1}{3}}$.

QUESTION 7

Given the function $f(x) = x^6 + 2x^5 + x^4$, let

A = The number guaranteed by Rolle's theorem on the interval [-1,0].
B = The number guaranteed by the mean value theorem for integrals on the interval [-1,1].
C = The value of the sixth derivative of $f(x)$ at $x = 4$.

Find $35B - 3A + \dfrac{D}{60}$.

QUESTION 8

An interesting property of partial derivatives is that they eliminate the need for implicit differentiation as we first learn it. A partial derivative is denoted as F_x and reads as the partial derivative with respect to x. To take a partial derivative of a multivariable function you simply treat the variable you are not differentiating with respect to as a constant. From multivariable calculus we know that $\frac{dy}{dx} = \frac{-F_x}{F_y}$ for any multivariable function. Use this to help you evaluate the following *(of course the method of implicit differentiation you first learn is still possible to use here, but is significantly more laborious in some cases).*

$$A = \frac{dy}{dx} \text{ given the function } x^2y + xy^2 - xy = 2.$$

$$B = \frac{dx}{dy} \text{ given the function } 2x\cos(4y) + x^3y^5 = 3x - e^{2xy}.$$

Find $\dfrac{A\Big|_{(x,y)=(1,\sqrt{2})}}{B\Big|_{(x,y)=(1,0)}}$.

QUESTION 9

A 17 foot ladder is propped up against a wall (the wall makes a right angle with the ground) and is sliding down the wall at a constant rate of 3 feet per second. Let $A =$ the rate of increase in the horizontal direction when the top of the ladder is 15 feet above the ground and on the wall.

A circle starts with radius of 2 meters at time $t = 0$. The area of the circle increases at a constant rate of 4 meters^2 per minute. Let B be equal to the radius of the circle when $t = 15\pi$ minutes.

Find $\dfrac{(A)(B)}{45}$.

QUESTION 10

$r = 3 + 3\sin(\theta)$

$$A = \text{The arc length of } r \text{ from } 0 \text{ to } \frac{\pi}{2}.$$

$$B = \text{The slope of the tangent line to } r \text{ at } \theta = \frac{\pi}{4}.$$

Find AB.

QUESTION 11

$$A = \text{The radius of convergence of } \sum_{n=1}^{\infty}\left((-1)^n \frac{(x-2)^n}{\sqrt{n}}\right).$$

$$B = \text{The radius of convergence of } \sum_{n=0}^{\infty}\left((-1)^n \frac{x^{2n+1}}{2n+1}\right).$$

$$C = \text{The radius of convergence of } \frac{1}{1-x}.$$

Find $\dfrac{A+B}{-C}$.

QUESTION 12

A curve, C, is defined by the parametric equations $x = t^3$ and $y = t^2 - 4$.

A = The interval of t on which C is monotonically increasing.

B = The minimum value of C.

C = The interval of t on which C is concave downward.

Find $A \cap C \cap (B, 4)$.

QUESTION 13

Let

$$\begin{aligned} A &= \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x \\ B &= \sum_{k=0}^{\infty}\frac{1}{k!} \\ C &= \left[\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right]^{-1} \end{aligned}$$

Given $\theta = A + B + C$, find $(\text{cis}(\theta))^{\frac{1}{ei}}$.

QUESTION 14

A ten foot long piece of wire can be cut once. The two pieces of metal remaining are bent into a square and an equilateral triangle.

A = The perimeter of the triangle when the area enclosed by the wire is a maximum.

B = The perimeter of the square when the area enclosed by the wire is a minimum.

Find AB.