For all questions, answer choice (E) NOTA means that none of the given answers is correct. i is defined as $\sqrt{-1}$. Let $cis\theta = \cos\theta + i\sin\theta$. Assume all inverse trigonometric functions have their traditional restricted ranges. Good Luck!

1. Convert $\frac{17\pi}{24}$ from radians to degrees.

   (A) $125°$ (B) $127.5°$ (C) $132°$ (D) $132.5°$ (E) NOTA

2. Find the sum of the values of x such that the matrix $\begin{bmatrix} 9 & 7 & x \\ 1 & 1 & 5 \\ x & 1 & 1 \end{bmatrix}$ does not have an inverse.

   (A) 24 (B) 34 (C) 36 (D) -36 (E) NOTA

3. Calculate $\sin 30° + \tan 45° - \cos 60°$.

   (A) $\sqrt{2}$ (B) $1+\sqrt{2}$ (C) 1 (D) $\frac{\sqrt{2}}{2}$ (E) NOTA

4. Compute $\arcsin\left(\cos\left(\arctan\left(-\sqrt{3}\right)\right)\right)$.

   (A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) NOTA

5. Find the dot product of $\langle 6, 5\rangle$ and $\langle -4, 2\rangle$.

   (A) 34 (B) 32 (C) -18 (D) -14 (E) NOTA

6. Which of the following is (are) even functions?

   I. $f(x) = 4 - \cos x$

   II. $f(x) = \sin x$

   III. $f(x) = 5 + \tan x$

   IV. $f(x) = \sec x$

   (A) I and II (B) II and III (C) II and III (D) I and IV (E) NOTA

7. What is the period of $f(x) = 3\tan\left(4x - \frac{\pi}{2}\right)$?

   (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) π (E) NOTA

8. Evaluate $\cos 15° + \sin 15° + \cos 75° - \sin 75°$.

   (A) $\frac{\sqrt{6}+\sqrt{2}}{4}$ (B) $\frac{\sqrt{2}-\sqrt{6}}{4}$ (C) $\frac{\sqrt{6}-\sqrt{2}}{4}$ (D) $\frac{-\sqrt{6}-\sqrt{2}}{4}$ (E) NOTA

9. Double the angle between the vectors $\langle 1, 1, 0\rangle$ and $\langle 2, 1, 2\rangle$. What is the sine of the resulting angle?

   (A) $\frac{1}{2}$ (B) 1 (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) NOTA

10. Mrs. Funk wrote a monic polynomial with real coefficients on the board. Jenny correctly found 2 of the roots, $1+5i$ and $2-3i$. Find Mrs. Funk's polynomial of minimal degree.

(A) $x^4-6x^3+42x^2-120x+338$ (B) $x^4-6x^3+47x^2-130x+338$
(C) $x^4+6x^3+47x^2-120x+338$ (D) $x^4-6x^3+42x^2-130x+338$ (E) NOTA

11. Find the length of the latus rectum of the ellipse $16x^2-128x+9y^2+18y+264=0$.

(A) 3 (B) $\frac{9}{2}$ (C) 4 (D) $\frac{3}{8}$ (E) NOTA

12. Express $\left(2e^{\frac{-25i\pi}{4}}\right)\left(\sqrt{2}e^{\frac{13i\pi}{3}}\right)$ in rectangular form.

(A) $1+\sqrt{3}-i+i\sqrt{3}$ (B) $1-\sqrt{3}-i+i\sqrt{3}$ (C) -1+$\sqrt{3}+i-i\sqrt{3}$ (D) -1+$\sqrt{3}-i+i\sqrt{3}$ (E) NOTA

13. Compute $\sum_{x=1^\circ}^{179^\circ}\cos x$.

(A) 2 (B) e (C) 1 (D) 0 (E) NOTA

14. Find the sum of the solutions of $\cot x = 2\sin x - \csc x$, where x is in the interval $[0, 2\pi)$.

(A) π (B) 2π (C) 3π (D) $\frac{5\pi}{2}$ (E) NOTA

15. The parametric equations $x=17\sin t+8$ and $y=17\cos t+3$ describe a curve in the Cartesian plane. What is the eccentricity of that curve?

(A) $\frac{8}{3}$ (B) $\frac{3}{8}$ (C) 1 (D) 0 (E) NOTA

16. Simplify $\dfrac{\operatorname{cis}\left(\frac{2\pi}{3}\right)}{\operatorname{cis}\left(\frac{\pi}{6}\right)}\times\left(\sin\left(\frac{\pi}{4}\right)+i\cos\left(\frac{3\pi}{4}\right)\right)$.

(A) $\operatorname{cis}\left(\frac{\pi}{4}\right)$ (B) $\operatorname{cis}\left(\frac{\pi}{6}\right)$ (C) $\operatorname{cis}\left(\frac{\pi}{3}\right)$ (D) $\operatorname{cis}\left(\frac{\pi}{5}\right)$ (E) NOTA

17. Compute RED, when $\dfrac{2x+3}{3x^3+8x^2+7x+2}=\dfrac{R}{3x+2}+\dfrac{E}{x+1}+\dfrac{D}{(x+1)^2}$.

(A) -75 (B) 15 (C) 75 (D) -15 (E) NOTA

18. What are the rectangular coordinates of the resulting point when $(-4\sqrt{3},12)$ is rotated $\frac{\pi}{2}$ counterclockwise about the origin?

(A) $(-12,-4\sqrt{3})$ (B) $(-12\sqrt{3},-4)$ (C) $(-8,-6\sqrt{3})$ (D) $(-8\sqrt{3},-6)$ (E) NOTA

19. Let a_n represent the nth term in the geometric sequence with 1st term, a_1, equal to 24 and the 50th term, a_{50} equal to 36. Compute $\sum_{n=1}^{50}\log_{864}(a_n)$.

(A) 20 (B) 22 (C) 25 (D) 27 (E) NOTA

20. Heronian triangles have integer side lengths and area. More specifically, almost-equilateral Heronian triangles have side lengths in the form $n-1$, n, and $n+1$. If Jenny has an almost-equilateral Heronian triangle of area 84, calculate the cosine of the angle opposite of the longest side.

(A) $\frac{2}{7}$ (B) $\frac{3}{7}$ (C) $\frac{4}{9}$ (D) $\frac{1}{3}$ (E) NOTA

21. Which of the following is (are) always true?

I. $\cot\theta + \tan\theta = \frac{2}{\sin(2\theta)}$

II. $\sin 31^\circ + \sin 29^\circ = 2\cos 30^\circ \sin 1^\circ$

III. $\cos 37^\circ - \cos 27^\circ = 2\sin 30^\circ \sin 7^\circ$

(A) I only (B) III only (C) I and II (D) I and III (E) NOTA

22. Find the solution(s) of $9^{\log x} \times x^{\log 9} - 2\left(9^{\log x} + x^{\log 9}\right) + 3 = 0$.

(A) 1 (B) 1 and .1 (C) 1 and $\sqrt{10}$ (D) 1 and $\sqrt[3]{10}$ (E) NOTA

23. Evaluate $(1+\cos 36^\circ)(\tan 18^\circ)(\sin 36^\circ)$. It is given that $\cos 36^\circ = \frac{1+\sqrt{5}}{4}$.

(A) $\frac{5+\sqrt{5}}{8}$ (B) $\frac{5-\sqrt{5}}{4}$ (C) $\frac{5+\sqrt{5}}{4}$ (D) $\frac{5-\sqrt{5}}{8}$ (E) NOTA

24. If the angle of rotation of $4x^2 + \sqrt{3}xy + 5y^2 + 6y + x = 11$ is β, what is $\sin(2\beta)$, given that $0 < \beta < \pi$

(A) 0 (B) $-\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1 (E) NOTA

25. How many times do the graphs of $r = \frac{1}{3\sin\theta + \cos\theta}$ and $r^2 = 25\sin 2\theta$ intersect?

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

26. Awnish has a pocket full of N pieces of candy. His pocket rips so that the pieces of candy can fall out. He knows if one piece of candy falls out then he could evenly divide the remaining pieces of candy with four other kids. However, if two pieces fall out then he could evenly divide the remaining pieces of candy with five other kids. What is the sum of the digits of the smallest possible value of N?

(A) 10 (B) 8 (C) 6 (D) 4 (E) NOTA

27. The points (6, 7, 6), (1, 2, 3), and (4, 3, 2) are on the plane $Ax - By + Cz = D$ when A, B, C, and D are all positive integers that do not share a common divisor. Compute $A + B - C - D$.

(A) 9 (B) 6 (C) 3 (D) 1 (E) NOTA

28. Convert the rectangular coordinates $(6, -2\sqrt{3})$ to polar coordinates.

(A) $\left(4\sqrt{3}, \frac{11\pi}{6}\right)$ (B) $\left(-3\sqrt{6}, \frac{11\pi}{6}\right)$ (C) $\left(4\sqrt{3}, \frac{\pi}{6}\right)$ (D) $\left(-3\sqrt{6}, \frac{\pi}{6}\right)$ (E) NOTA

29. Which of the following is a root of $f(x) = x^6 + x^3 + 1$?

(A) $\text{cis}\left(\frac{2\pi}{3}\right)$ (B) $\text{cis}\left(\frac{4\pi}{3}\right)$ (C) $\text{cis}\left(\frac{4\pi}{9}\right)$ (D) $\text{cis}\left(\frac{\pi}{2}\right)$ (E) NOTA

30. Siddarth is in love with the soccer star, Lionel Messi. Jenny, the rather notorious matchmaker, tells Siddarth that she will arrange a date with Messi for him if he can answer the following question.

$$\text{Given that } \sin\theta + \cos\theta = \frac{1}{2}, \text{ evaluate } \tan^3\theta + \cot^3\theta.$$

Help Siddarth go on his dream date. What is the answer to Jenny's question?

(A) $-\frac{286}{27}$ (B) $-\frac{296}{27}$ (C) $-\frac{316}{27}$ (D) $-\frac{326}{27}$ (E) NOTA