For all questions, answer choice (E) NOTA means that none of the given answers is correct. Assume that all inverse trigonometric functions have their traditional restricted ranges. Let $i = \sqrt{-1}$. Good Luck!

1. The points $(-2, 5)$, $(-1, 3)$, and $(-3, 1)$ are three of the four vertices of a parallelogram. Given that the fourth vertex lies in the second quadrant, find the sum of the fourth vertex's x- and y-coordinates.

   (A) -2 (B) -1 (C) 0 (D) 1 (E) NOTA

2. The graph of $f(x)$ is a parabola. Given that $f(-1) = 3$, $f(0) = 2$, and $f(1) = 5$, find $f(7)$.

   (A) 93 (B) 100 (C) 107 (D) 110 (E) NOTA

3. Given that $2^x = 3^y$, find $\dfrac{y}{x}$.

   (A) $\dfrac{2}{3}$ (B) $\log \dfrac{2}{3}$ (C) $\ln \dfrac{2}{3}$ (D) $\log_3 2$ (E) NOTA

4. At time $t = 0$, a ladybug is at $(-3, 7)$, and a june bug is at $(0, 1)$. The ladybug begins traveling in the direction of $(2, 3)$, and the june bug begins traveling in the direction of $(1, -1)$. Find the sum of the coordinates of the point where the bugs meet.

   (A) 5 (B) $-\dfrac{3}{2}$ (C) $\dfrac{5}{2}$ (D) -3 (E) NOTA

5. Find $\cot^{-1}\left(\cos\left(\tan^{-1}\left(\csc\left(\tan^{-1}\left(\sin\left(\dfrac{\pi}{2}\right)\right)\right)\right)\right)\right)$.

   (A) $\dfrac{-\pi}{3}$ (B) $\dfrac{-\pi}{6}$ (C) $\dfrac{\pi}{6}$ (D) $\dfrac{\pi}{3}$ (E) NOTA

6. Drake's path can be modeled by the function $y = x^4 - 8x^3 + 10x^2 + 48x - 23$. Given that Drake started from the bottom of the function (or, the global minimum), find the y-coordinate of his starting point.

   (A) 73 (B) -30 (C) -43 (D) -52 (E) NOTA

7. Evaluate $\displaystyle\int_{-3}^{3\sqrt{2}} \sqrt{36 - x^2}\, dx$.

   (A) $\dfrac{18 + 15\pi + 9\sqrt{3}}{2}$ (B) $\dfrac{18 + 15\pi - 9\sqrt{3}}{2}$ (C) $\dfrac{18 + 9\pi + 9\sqrt{3}}{2}$ (D) $\dfrac{18 + 9\pi - 9\sqrt{3}}{2}$ (E) NOTA

8. Billy and Jimmy are playing a game. Billy rolls two fair, six-sided dice. His score is the sum of the values shown on the dice. Jimmy flips two fair coins. His score is 5 points for each head and 2 points for each tail. What is the probability of Billy scoring more points than Jimmy?

   (A) $\dfrac{3}{8}$ (B) $\dfrac{7}{16}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{9}{16}$ (E) NOTA

9. The number 12345AB, where A and B are integers between 0 and 9 inclusive, is divisible by 22. How many possible combinations (A, B) exist?

   (A) 0 (B) 3 (C) 4 (D) 5 (E) NOTA

10. This 5×5 grid contains 5 instances of every number in the set 1, 2, 3, 4, 5. Each number occurs once in every row, and once in every column. Find the sum of the numbers on the main diagonal of the grid (running from the top left corner to the bottom right corner).

4				5
			1	
	2			3
		3	5	
1				

(A) 14 (B) 15 (C) 16 (D) 18 (E) NOTA

11. A triangle has side lengths of 5, 12 and 13. Two circles are drawn, one which inscribes the triangle, and one which circumscribes it. Find the positive difference between their areas.

(A) $\frac{119\pi}{4}$ (B) 32π (C) 36π (D) $\frac{153\pi}{4}$ (E) NOTA

12. Find the sum of the coordinates of the removable discontinuity of $\frac{x^3 - 8x^2 + x + 42}{x - 3}$.

(A) 23 (B) 17 (C) -17 (D) -23 (E) NOTA

13. Find the domain of $\ln\left(\sqrt{\log_5 3x}\right)$

(A) $(-\infty, \infty)$ (B) $(0, \infty)$ (C) $\left(\frac{1}{3}, \infty\right)$ (D) $\left(\frac{5}{3}, \infty\right)$ (E) NOTA

14. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 & 7 \end{bmatrix} = \begin{bmatrix} 10 & 12 & 14 \\ 15 & 18 & 21 \end{bmatrix}$, what is $A^2 + B^2$?

(A) $\frac{13}{4}$ (B) 5 (C) $\frac{25}{4}$ (D) 8 (E) NOTA

15. $\cos(2\theta) + 3 = \sin(2\theta)\tan(\theta)$. Find the sum of the solutions on the interval $[0, 2\pi)$.

(A) $\frac{3\pi}{2}$ (B) 2π (C) $\frac{7\pi}{2}$ (D) no solutions exist (E) NOTA

16. If you got a problem, yo, Vanilla Ice will solve it. Well, Jay-Z has 99 problems. Mr. Ice has agreed to solve these problems, but he cannot do so very fast. It takes him n^2 minutes to solve the nth problem. How many minutes can Jay-Z expect to wait for Mr. Ice to solve all of his problems?

(A) 304950 (B) 328350 (C) 357150 (D) 388650 (E) NOTA

17. Let $f(x)$ be a differentiable function such that

$$\begin{aligned} f'(x) &= f(x) \\ f(2) &= 10 \end{aligned}$$

Find $f(4)$.

(A) $10e^4$ (B) 10 (C) $e^4 + 10$ (D) $e^2 + 10$ (E) NOTA

18. A line segment is drawn from the origin to a randomly selected point in the first quadrant. Find the probability that $1 < m < 2$, where m represents the slope of the line segment.

(A) $\dfrac{2\sin^{-1}\left(\dfrac{\sqrt{5}}{5}\right)}{\pi}$ (B) $\dfrac{2}{\pi}$ (C) $\dfrac{2\sin^{-1}\left(\dfrac{\sqrt{5}}{2}\right)}{\pi}$ (D) $\dfrac{2\sin^{-1}\left(\dfrac{\sqrt{10}}{10}\right)}{\pi}$ (E) NOTA

19. How many of the following numbers are transcendental?

$$e,\ \pi,\ \sqrt{3},\ 1+2i,\ 2013,\ \phi$$

(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

20. For every positive integer n, there exists a Pythagorean Triple whose largest element is 5^n, and whose smaller two elements are not divisible by 5. Find this triple for $n = 3$, and give the sum of the two smaller elements. (Hint: $3 + 4i$)

(A) 175 (B) 151 (C) 161 (D) 31 (E) NOTA

21. The circle with equation $x^2 + y^2 = 1$ is graphed in the coordinate plane. Let A and B be the two points on the circle with x-coordinate $\dfrac{1}{10}$. The tangent lines to the circle at points A and B intersect at point C. What is the shortest distance from point C to the circle?

(A) 9 (B) $3\sqrt{10}$ (C) $\dfrac{99}{10}$ (D) 10 (E) NOTA

22. $\sqrt{30 - 12\sqrt{6}} = a\sqrt{b} + c\sqrt{d}$, where a, b, c, and d are integers such that b and d are not divisible by the square of any prime. Find $a + b + c + d$.

(A) 4 (B) 6 (C) 9 (D) 12 (E) NOTA

23. $\displaystyle\sum_{n=1}^{z} \lfloor \log_3 n \rfloor = 150$, where $\lfloor x \rfloor$ equals the largest integer less than or equal to x. Find z.

(A) 60 (B) 61 (C) 62 (D) 63 (E) NOTA

24. Solve for a in the following system of equations:

$$\begin{aligned} 2a + 4b - 3c &= -1 \\ 3a + b + c &= 2 \\ a + 5b - 2c &= 3 \end{aligned}$$

(A) -2 (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) 2 (E) NOTA

25. Evaluate $\cos\left(\dfrac{2\pi}{7}\right) + \cos\left(\dfrac{4\pi}{7}\right) + \cos\left(\dfrac{6\pi}{7}\right)$.

(A) 0 (B) $-\dfrac{1}{2}$ (C) $-\dfrac{1}{3}$ (D) $-\dfrac{2}{3}$ (E) NOTA

26. Kendrick is filling his swimming pool. He can fill it in 10 hours with the drain closed, but the drain can empty the pool in 30 hours. If the drain is currently open, and he fills the pool in 12 hours, how long does he have to close the drain in order to fill the pool?

(A) 3 hours, 36 minutes (B) 4 hours (C) 6 hours (D) 5 hours, 24 minutes (E) NOTA

27. There exist two parabolas, $f(x)$ and $g(x)$. $f(x) = -(x-3)^2 + 5$ and $g(x) = -(x-h)^2 + k$, where h and k are real numbers. The tangent line to $f(x)$ at $(5, 1)$ is the normal line to $g(x)$ at the same point. Find $h + k$.

(A) $\frac{351}{64}$ (B) $\frac{365}{64}$ (C) $\frac{381}{64}$ (D) $\frac{393}{64}$ (E) NOTA

28. $f(x) = ax^2 + bx + c$. a is a randomly selected number between -3 and 4. b is a randomly selected number between -5 and 7. Find the probability that the vertex of $f(x)$ lies to the left of the y-axis.

(A) $\frac{41}{84}$ (B) $\frac{43}{84}$ (C) $\frac{5}{12}$ (D) $\frac{7}{12}$ (E) NOTA

29. Find the positive difference between the smallest positive integer with 11 factors and the smallest positive integer with 12 factors.

(A) 904 (B) 964 (C) 1624 (D) 1712 (E) NOTA

30. An aardvark, a badger, a chinchilla, and a dalmatian are all participants in a fashion show. Each animal has its own unique prop: one has a sweater, another has a scarf, another has a sombrero, and another has sunglasses. One prop is pink, another is red, another is blue, and another is green. The sombrero is green. The badger is wearing red. Neither the chinchilla nor the aardvark is wearing the sweater. The chinchilla is wearing pink. The blue scarf is not being worn by the dalmatian. Which animal is wearing the sunglasses?

(A) The aardvark (B) The badger (C) The chinchilla (D) The dalmatian (E) NOTA