QUESTION 1

Let

$$\begin{aligned}
A &= \lim_{x\to\infty} \frac{3x^2+2x+1}{x^2+2x+7} \\
B &= \lim_{x\to 3} \frac{x^2+x-12}{x-3} \\
C &= \lim_{x\to 0} \frac{\sin x}{x} \\
D &= \lim_{x\to 0} \frac{\cos x - 1}{x}
\end{aligned}$$

Find $A+B+C+D$.

QUESTION 2

Let

$$\begin{aligned}
A &= f'\left(\frac{\pi}{4}\right) \text{ if } f(x) = \frac{\tan(x) - 1}{\sec(x)} \\
B &= \text{the positive value of } \left.\frac{dx}{dy}\right|_{y=6} \text{ if } y = x^2 + 5 \\
C &= D_x y\Big|_{x=\frac{\pi}{6}} \text{ if } y = \sec x \\
D &= \left.\frac{d^2y}{dx^2}\right|_{x=0} \text{ if } y = \arctan x
\end{aligned}$$

Find $A + D + BC$.

QUESTION 3

Let

$$\begin{aligned} A &= \int_1^2 x^{-3} + x^4 \, dx \\ B &= \int_1^\infty \frac{1}{x^2} \, dx \\ C &= \frac{d}{dx} \int_0^\pi (\cos t - \sin t) \; dt \\ D &= \text{The arc length of the curve } y = \frac{2x^{\frac{3}{2}}}{3}, \text{ from } x = 1 \text{ to } x = 2 \end{aligned}$$

Find $AB + CD$.

QUESTION 4

A = the velocity of a particle at $t = 2$, given that $a(t) = t^3$ and $v(0) = 4$

B = the average value of $f(x) = x \sin x$, over the interval $\left[\frac{\pi}{2}, \pi\right]$

C = The volume of the solid with semicircular cross sections whose base is defined by the unit circle

D = The volume of the solid obtained by rotating the area bounded by $y = \sqrt{x}$, $y = 3$, and the y-axis about the y-axis

Find $A + \pi B + 3C + 5D$.

QUESTION 5

Consider the function, $f(x) = x(3x + 7)$ for this question.
Let

$$\begin{aligned} A &= f'(3) \\ B &= \int_0^{10} f(x)\,dx \\ C &= \text{The number of local maxima that } f(x) \text{ has} \\ D &= \text{the average rate of change of } f(x) \text{ over the interval } [1,4] \end{aligned}$$

Find $A + B + C + D$.

QUESTION 6

Let

$$\begin{aligned}
A &= \sum_{n=1}^{\infty} \frac{5^n - 3^n}{7^n} \\
B &= \text{the radius of convergence of } \sum_{n=1}^{\infty} \frac{2^n}{n}(4x-8)^n \\
C &= f\left(\frac{\pi}{3}\right),\ \text{if } f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \\
D &= f(\ln 5),\ \text{if } f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}
\end{aligned}$$

Find $ABCD$.

QUESTION 7

Let the *chaoticity* of a system A be denoted as S. Let the chaoticity be defined such that if a system has a heat of Q_1, then the change in chaoticity of the system when it possesses a new heat of Q_2 is equal to

$$\Delta S = \int_{Q_1}^{Q_2} \frac{c}{Q}\, dQ$$

where c is some real number. If a system is composed of multiple objects, define the chaoticity change to be the sum of the chaoticity changes of each object. A system is at *thermal equilibrium* if all objects have the same heat. Assume the total heat of a system never changes. Then let

A = the value of $\dfrac{Q_2}{Q_1}$ if $\Delta S = 0$

B = the value of c such that $\Delta S = 10$ when a system's heat goes from 4 to 20

C = the value of ΔS if A is composed of two objects with initial heats 4 and 5 and $c = 1$, but then goes to thermal equilibrium

D = the maximum possible value of ΔS if A is composed of two objects with initial heats 4 and 5 and $c = 1$

Find $A + e^{\frac{10}{B}} + \dfrac{C + D}{2}$.

QUESTION 8

Let

$$\begin{aligned} A &= \text{the area bounded between the curves } y = x^2 \text{ and } y = -x^2 + 4 \\ B &= \text{the maximum area of a rectangle with vertices on the origin, the } x \text{ and } y \text{ axes, and the curve } e^{-x^2} \\ C &= \text{the second approximation of a root of } y = x^2 - 4x + 2 \text{ with initial guess } x_0 = 3 \end{aligned}$$

Find $AB^2 + 4C$.

QUESTION 9

Let

$$\begin{aligned} A &= \frac{dy}{dx} \text{ at } x = 1, \text{ if } x^2 + 2x + xy = 6 \\ B &= y(1), \text{ if } \frac{dy}{dx} = 2x \text{ and } y(0) = 2 \\ C &= g(3), \text{ if } g(x) = F(x)F\left(\frac{1}{x}\right) \text{ where } F(x) \text{ is an antiderivative of a function } f(x) \text{ such that } F(x)f\left(\frac{1}{x}\right) = x \text{ for} \\ &\quad \text{all } x \text{ in the domain of } f(x) \text{ and } f(1) = 2 \end{aligned}$$

Find ABC.

QUESTION 10

Sand is being poured onto a conical pile at a constant rate of 50 cm^3/s. Frictional forces require that the height of the cone always be equal to its radius. Let A be the rate of change of the height when the height of the pile is 5 cm.

Consider a triangle with two legs of length 15 and 20, such that the angle between them is changing at a constant rate of $\frac{\pi}{90}$ radians per second. Let B be the rate of change of the third side when the angle has a measure of $\frac{\pi}{3}$.

Find AB.

QUESTION 11

Let $f(x)$ be a continuous function that is increasing and concave up on the interval $[a, b]$ for some real numbers a and b. Then for some positive integer n, let

$$M = \text{the approximation of } \int_a^b f(x)\,dx \text{ using the Midpoint Rule on } n \text{ equal subintervals}$$

$$S = \text{the approximation of } \int_a^b f(x)\,dx \text{ using Simpson's Rule on } 2n \text{ equal subintervals}$$

$$T = \text{the approximation of } \int_a^b f(x)\,dx \text{ using the Trapezoid Rule on } n \text{ equal subintervals}$$

$$L = \text{the approximation of } \int_a^b f(x)\,dx \text{ using a Left Riemann sum on } n \text{ equal subintervals}$$

$$R = \text{the approximation of } \int_a^b f(x)\,dx \text{ using a Right Riemann sum on } n \text{ equal subintervals}$$

Order the approximations in increasing order of magnitude.

QUESTION 12

Let

$$\begin{aligned} A &= \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{\frac{1}{n}} \\ B &= \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{n} \\ C &= \lim_{n\to\infty} \frac{\sqrt[n]{\frac{(2n)!}{n!}}}{n} \end{aligned}$$

Find ABC.

QUESTION 13

Each of the statements below has an associated point value. Find the sum of the point values of the statements that are true.

1 = If for two functions $f(x)$ and $g(x)$, we have that $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x) = \infty$, then $\lim_{x\to 0}[f(x) - g(x)] = 0$.

-5 = If $f(x)$ and $g(x)$ are both differentiable, then $\frac{d}{dx}[f(x)g(x)] = f'(x)g'(x)$.

3 = If for some twice-differentiable function $f(x)$ we have $f''(2) = 0$, then $(2, f(2))$ is an inflection point of the curve $y = f(x)$.

7 = All continuous functions have antiderivatives.

5 = If $f(x) \leq g(x)$ for all x and $\int_0^\infty g(x)\,dx$ diverges, then $\int_0^\infty f(x)\,dx$ also diverges.

-10 = If for some sequence $\{a_n\}_{n=0}^\infty$, $\sum_{n=0}^\infty a_n$ is divergent, then $\sum_{n=0}^\infty |a_n|$ is also divergent.

QUESTION 14

Let

$$\begin{aligned} A &= \text{the slope of the tangent of } \ln|\sec x + \tan x| \text{ at } x = \pi \\ B &= \int_0^{\frac{\pi}{3}} \sec^3 x \, dx \end{aligned}$$

Find AB.