For all questions, answer choice (E) NOTA means that none of the given answers for a question are correct. On this test, $i = \sqrt{-1}$. Assume all inverse trigonometric functions have their traditional restricted ranges. Additionally, cis(θ) is defined as $\cos\theta + i\sin\theta$. Good luck!

1. Convert $\frac{11\pi}{30}$ from radians to degrees.

   (A) 33° (B) 66° (C) 132° (D) 22° (E) NOTA

2. Evaluate $\cos 135^\circ + \sin 75^\circ + \tan 225^\circ$.

   (A) $\frac{4+\sqrt{6}-\sqrt{2}}{4}$ (B) $\frac{4+\sqrt{6}+\sqrt{2}}{4}$ (C) $\frac{-4+\sqrt{6}-\sqrt{2}}{4}$ (D) $\frac{-4+\sqrt{6}+\sqrt{2}}{4}$ (E) NOTA

3. Evaluate $\begin{vmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{vmatrix}$.

   (A) 1 (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) 0 (E) NOTA

4. The expression $\frac{\sin 2\theta \tan\theta}{2}$, where $0 < \theta < \frac{\pi}{2}$, is equivalent to which of the following expressions?

   (A) $\frac{\cos^2\theta}{2}$ (B) $\frac{\sin^2\theta}{2\cos\theta}$ (C) $\sin^2\theta$ (D) $\cos 2\theta$ (E) NOTA

5. Compute the dot product of the vectors $\langle 1, 2, 6\rangle$, $\langle -2, 1, -4\rangle$.

   (A) 4 (B) 12 (C) -24 (D) 0 (E) NOTA

6. Evaluate $\left|\sec\frac{2\pi}{3} + i\csc\frac{2\pi}{3}\right|$.

   (A) $\frac{4\sqrt{3}}{3}$ (B) $\frac{6+2\sqrt{3}}{3}$ (C) 2 (D) $\frac{6-2\sqrt{3}}{3}$ (E) NOTA

7. Determine the focus of the parabola $y = \frac{1}{4}(x-7)^2 + 3$.

   (A) $(8, 4)$ (B) $(7, 4)$ (C) $(7, 3)$ (D) $(7, 2)$ (E) NOTA

8. Which of the following matrices A satisfies the property $A^2 = I$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$?

   (A) $\begin{bmatrix} \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ (E) NOTA

9. Find the sum of the distinct solutions of $\cos 2x + \cos x = 0$, where x is in the interval $[0, 2\pi)$.

   (A) π (B) 2π (C) 3π (D) 4π (E) NOTA

10. Find the area of the conic satisfying the following equation:

$$4x^2 + 9y^2 - 32x + 54y + 73 = 0$$

   (A) 6π (B) 12π (C) 18π (D) 24π (E) NOTA

11. Consider a triangle $\triangle ABC$, such that $\angle BAC = 60°$, and $AB = 8, AC = 10$. Compute the length of BC.

(A) $2\sqrt{41 - 20\sqrt{3}}$ (B) $\sqrt{21}$ (C) 8 (D) $2\sqrt{21}$ (E) NOTA

12. Matthew likes to eat red velvet cheesecake, and one day discovers a local cheesecake shop that has great red velvet cheesecake, and decides to visit it every 8 days, starting from that day. On that same day, Mihir happens to be visiting that same shop and discovers that it has great free WiFi, and so thus decides to visit this shop every 6 days starting from that day. Over the period of the next 60 days (not counting that first day), how many times do they visit the shop on the same day?

(A) 4 (B) 3 (C) 2 (D) 1 (E) NOTA

13. Find the product of all distinct real x satisfying the equation:

$$(\ln x)^2 + \ln x^2 + 1 = 0$$

(A) 1 (B) e (C) $\frac{1}{e}$ (D) $\frac{1}{e^2}$ (E) NOTA

14. Consider two real numbers a, b such that $ab = 2$ and $a^2 + 3a + 3 = b$. Determine the numerical value of a.

(A) $-1 + \sqrt[3]{3}$ (B) $\sqrt[3]{3}$ (C) $-1 + \sqrt[3]{2}$ (D) $1 + \sqrt[3]{3}$ (E) NOTA

15. Let the point (x, y) be the removable discontinuity of the function

$$y = \frac{x^3 + 3x^2 - 18x - 40}{x^2 - 7x + 12}$$

Compute $x + y$.

(A) 4 (B) 58 (C) 74 (D) $\frac{82}{7}$ (E) NOTA

16. Compute the length of the tangent segment from the point $(1, 2)$ to the circle whose equation is $(x+2)^2+(y+2)^2 = 9$.

(A) 5 (B) 4 (C) 16 (D) $\sqrt{34}$ (E) NOTA

17. The expression $1 + (\log_3 2)(\log_3 18)$ is equivalent to which of the following?

(A) $\log_3 108$ (B) $\log_3 18$ (C) $1 + (\log_3 2)^2$ (D) $(\log_3 6)^2$ (E) NOTA

18. Let α, β be acute angles such that $\sin\alpha = \frac{3}{5}$ and $\sin\beta = \frac{5}{13}$. Compute $\cos(\alpha - \beta)$.

(A) $\frac{33}{65}$ (B) $-\frac{8}{65}$ (C) $\frac{8}{65}$ (D) $\frac{63}{65}$ (E) NOTA

19. Let r_1, r_2, r_3 be the roots of the cubic polynomial $x^3 - 4x^2 - 19x - 14$. Compute the value of $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$.

(A) $-\frac{1}{4}$ (B) $-\frac{19}{14}$ (C) $\frac{1}{4}$ (D) $\frac{2}{7}$ (E) NOTA

20. If $\frac{13x + 7}{3x^2 - x - 2} = \frac{A}{x - 1} + \frac{B}{3x + 2}$ and A, B are real numbers, then compute $A^2 + B^2$.

(A) 5 (B) 10 (C) 17 (D) 25 (E) NOTA

21. Find the sum of the coefficients in the simplified binomial expansion of $(x + iy)^{10}$.

(A) $32i$ (B) $16 + 16i$ (C) $16 - 16i$ (D) 32 (E) NOTA

22. Consider a triangle $\triangle ABC$ and point D on the segment $\overline{BC}$ such that $BD = 3$ and $CD = 1$. Given that $AB = 4$ and $AC = 5$, evaluate $\dfrac{\sin \angle BAD}{\sin \angle CAD}$.

(A) $\dfrac{15}{4}$ (B) $\dfrac{5}{12}$ (C) $\dfrac{12}{5}$ (D) 3 (E) NOTA

23. Let a be a complex number such that $a + \dfrac{1}{a} = 2\cos 18^\circ$. If $a^3 + \dfrac{1}{a^3} = 2\cos b^\circ$, where b is an integer such that $0 \le b \le 90$, compute the product of the digits of b.

(A) 8 (B) 14 (C) 18 (D) 20 (E) NOTA

24. If a, b, c are the roots of the cubic polynomial $6x^3 - 13x + 17$, compute the value of $a^3 + b^3 + c^3 - 3abc$.

(A) $\dfrac{10}{3}$ (B) $\dfrac{17}{2}$ (C) $-\dfrac{17}{2}$ (D) 0 (E) NOTA

25. Let x be a real number. Then evaluate

$$\begin{vmatrix} x+0.5 & x \\ x+1 & x+0.5 \end{vmatrix} + \begin{vmatrix} x+0.5 & x \\ x+1 & x+0.5 \end{vmatrix}^2 + \begin{vmatrix} x+0.5 & x \\ x+1 & x+0.5 \end{vmatrix}^3 + \cdots$$

(A) $\dfrac{4}{3}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{5}{4}$ (D) $\dfrac{1}{2}$ (E) NOTA

26. One day, a very bored Jenny decided to make an extremely amusing post on her Friend's wall. However, 3 other friends stumbled upon this message and thinking it to be too amusing not to share, decide to spread this to others. The next day, 7 new people find out. From every day forward, if on some day, n new people find out about this message, then on the next day, $3n - 2$ find out about it. If the day when the 3 friends stumble upon the message is day 1, how many people find out about the message on the tenth day? (Hint: Notice the sequence goes $3, 7, 19$; try to find a pattern)

(A) $3^{10} - 2$ (B) $3 + 4 \cdot 3^9$ (C) $1 + 2 \cdot 3^9$ (D) $3^{10} - 2^{10}$ (E) NOTA

27. Let a be the amplitude of the trigonometric function $f(x) = \sin x + \sqrt{3}\cos x$, and let b be the minimum value in the interval $[0, 2\pi]$ such that $f(b) = 0$. Compute ab.

(A) $-\dfrac{2\pi}{3}$ (B) $\dfrac{4\pi}{3}$ (C) $\dfrac{10\pi}{3}$ (D) $\dfrac{2\pi}{3}$ (E) NOTA

For questions 28-30, use the following information:
A complex number z is called an n**th root of unity** if $z^n = 1$, given that n is a positive integer. For any n, there are exactly n distinct roots of unity. z is an n**th primitive root of unity** if n is the minimum positive integer such that $z^n = 1$.

28. If x is a root of the polynomial $x^2 + x + 1$, what is the smallest positive integer n such that x is an nth root of unity?

(A) 2 (B) 4 (C) 6 (D) 8 (E) NOTA

29. Which of the following numbers are 6th primitive roots of unity?

$$\text{I. } \operatorname{cis}\left(\frac{\pi}{2}\right) \qquad \text{II. } \operatorname{cis}\left(\frac{5\pi}{6}\right)$$
$$\text{III. } \operatorname{cis}\left(\frac{\pi}{3}\right) \qquad \text{IV. } \operatorname{cis}\left(\frac{5\pi}{3}\right)$$

(A) III, IV (B) I, III, IV (C) II only (D) I, II, III, IV (E) NOTA

30. Let a be the number of distinct 24th primitive roots of unity. Let b be the number of positive integers n such that there are exactly 5 nth primitive roots. Find $a + b$.

(A) 10 (B) 29 (C) 8 (D) 13 (E) NOTA