Question 1

Let

$$
\begin{aligned}
A &= \text{the value of } x \text{ at which the function } r(x) = \frac{2x^2 - 7x + 6}{3x^2 - 5x - 2} \text{ has an infinite discontinuity} \\
B &= \text{the average rate of change of the function } s(x) = \log_2(x^2 + 10x + 8) \text{ over the interval } [0, 2] \\
C &= g'(3), \text{ given that } f(x) = (x-2)^3 + 4 \text{ and } g(x) = f^{-1}(x) \\
D &= \text{the value of the integral } \int_0^{\frac{\pi}{2}} \frac{x\cos x - \sin x}{x^2}\, dx
\end{aligned}
$$

Find $A + B + C + D$

Question 2

Let

$$
\begin{aligned}
A &= \text{the maximum possible value of the product of two real numbers, given that their sum is 10}\\
B &= \text{the minimum possible distance from the point } (5,0) \text{ to a point on the curve } f(x) = x^2 + 1\\
C &= \text{the maximum value of the function } h(x) = x^3 - 6x^2 + 9x + 10 \text{ on the closed interval } [0,5]\\
D &= \text{the largest possible area of a triangle inscribed in the region bounded by the graphs of}\\
&\quad f(x) = -x^2 + 6x \text{ and } g(x) = 2x \text{ (that is, every point in the interior of the triangle is also in the}\\
&\quad \text{interior of the given region)}
\end{aligned}
$$

Find $A + B + C + D$

Question 3

Let

A = the slope of the line passing through the origin and tangent to the graph of $f(x) = x^2 + 5$, where $x > 0$.

B = the approximation of $\sqrt{2012}$ using the line tangent to the curve $f(x) = \sqrt{x}$ at the point $(2025, 45)$.

C = the area of the triangular region bounded by the coordinate axes and the line tangent to the graph of $g(x) = \dfrac{1}{x}$ at $x = 2$.

Find $A + B + C$

Question 4

For this problem, give only the answer to part D. In this problem, you will evaluate an integral using the substitution $t = \tan\left(\frac{\theta}{2}\right)$.

$$
\begin{aligned}
A &= \sin\left(\frac{\theta}{2}\right) \text{ and } \cos\left(\frac{\theta}{2}\right) \text{ in terms of } t \\
B &= \sin\theta \text{ in terms of } t \\
C &= \cos\theta \text{ in terms of } t \\
D &= c\text{, where} \\
& \int_0^{\frac{\pi}{2}} \frac{1}{\sin\theta + \sqrt{3}\cos\theta}\, d\theta = \frac{1}{2}\ln c
\end{aligned}
$$

Question 5

Consider the function $f(x) = x(x-3)^3$. Then let

$$\begin{aligned} A &= \text{the value of } f'(0) \\ B &= \text{the length of the interval over which } f(x) \text{ is concave down} \\ C &= \text{the value of } x \text{ at which } f(x) \text{ has a local minimum} \\ D &= \text{the number of local maxima that } f(x) \text{ has} \end{aligned}$$

Find $A + B + C + D$

Question 6

Let

$$\begin{aligned} A &= \text{the volume of the solid formed by rotating the region bounded by } y = x^3,\ y = 8, \text{ and } x = 0 \\ &\quad \text{about the } y\text{-axis} \\ B &= \text{the volume of the solid formed by rotating the region bounded by } y = 2x^2 - x^3 \text{ and } y = 0 \\ &\quad \text{about the } y\text{-axis} \\ C &= \text{the volume of the solid with square cross sections whose base is defined by the equation} \\ &\quad x^2 + y^2 = 16 \\ D &= \text{the area bound by the graph of } y = -(x-2)^2 + 4 \text{ and the } x\text{-axis} \end{aligned}$$

Find $A + B + C + D$

Question 7

Let

$$\begin{aligned} A &= \lim_{x \to 0^+} x \ln\left(\frac{1}{x}\right) \\ B &= \lim_{x \to -1} \frac{3x^2 + 8x + 5}{x + 1} \\ C &= \lim_{x \to \infty} \frac{2e^x + 4x + 6}{e^x + 6x + 3} \end{aligned}$$

Find $A + B + C$

Question 8

List the letters of all statements below which are true. If none are true, answer "None."

A = A differential of a single-variable function is itself a function.

B = When using Newton's method to approximate a zero of a function, any value of the initial guess will converge to the same result.

C = If $f(x)$ is any real-valued, single-variable, twice-differentiable function with domain $\mathbb{R}$ such that $f(0) > 0$ and is concave down on its entire domain, then $f(x)$ has at least one root.

D = All polynomials in one variable are infinitely differentiable with respect to that variable.

E = If $f'(x) = g'(x)$ one some interval then $f(x) = g(x) + C$ on that interval for some constant C.

F = A function has either a local minimum or local maximum wherever its derivative equals zero.

Question 9

Let

$$\begin{aligned} A &= f'\left(\frac{1}{2}\right), \text{ if } f(x) = \cos\left(4\cos^{-1}(x)\right) \\ B &= g'\left(\frac{1}{2}\right), \text{ if } g(x) = \sum_{n=1}^{\infty} nx^n \text{ for } -1 < x < 1. \\ C &= \text{the slope of the line tangent to the curve } (x-3)^2\left(x^2+y^2\right) = 4x^2 \text{ at the point } (2, 2\sqrt{3}) \end{aligned}$$

Find $A + B + C$

Question 10

A laser pointer, held at a constant position parallel to the ground, is rotating counterclockwise such that the resulting dot on a wall 10 feet away moves at a constant 3 ft/sec. A will be the the rate at which the laser pointer is turning in rad/sec when the dot is 20 feet away from the laser pointer.

By evaluating the following integral, find the value of k. B will be the value of k.

$$\int_1^2 \frac{1}{x^3 + x}\, dx = \ln k$$

Find AB^2

Question 11

Let p, q be real numbers, and let $f(x)$ be a function such that

$$f(x) = \begin{cases} x^2, \text{ if } x \leq 1 \\ x^3 + px^2 + qx, \text{ if } x > 1 \end{cases}$$

Then let

$$\begin{aligned} A &= p + q, \text{ if } f(x) \text{ is continuous} \\ B &= q, \text{ if } f(x) \text{ is differentiable} \end{aligned}$$

Find $A + B$

Question 12

Matt is pouring water into an initially empty cup in the shape of a cone at a constant rate of $3\pi\,\text{cm}^3/\text{sec}$. The cone has a radius of 4 cm and height 20 cm. Let A be the rate at which the water level is rising in cm/sec after 15 seconds.

A constant force is applied to an initially resting object over a period of 20 seconds, at the end of which the object is traveling 100 meters/second. Assuming no friction, let B be the distance that the object traveled in meters during that period.

Find $\dfrac{B}{A}$

Question 13

If $f(x)$ is a differentiable function such that $f'(x) = 2f(x)$ and $f(0) = 3$, let $A = f(5)$.

If $g(x)$ is a twice-differentiable function such that $g''(x) = 1$, $g'(0) = 2$, and $g(0) = 3$, let $B = g(5)$.

If $r(x)$ is a differentiable function such that $r'(x) = -\sin x$ and $r(0) = 3$, let $C = r\left(\frac{\pi}{3}\right)$.

Find $A + 4BC$

Question 14

Let

$$
\begin{aligned}
A &= \text{the area of the region enclosed by the graph of the polar function } r = 4\sin\theta \\
B &= \text{the area of the region enclosed by } x\text{-axis and the function } f(x) = x^2 - 2x - 9
\end{aligned}
$$

Find $\dfrac{B}{A}$