

For all questions, choice E: NOTA means that none of the given answers is correct. Figures are not necessarily drawn to scale. Good Luck!

1. Convert $(55^\circ + 70^\circ + 115^\circ)$ to radians.

(A) $\frac{2\pi}{3}$ (B) $\frac{4\pi}{5}$ (C) $\frac{4\pi}{3}$ (D) $\frac{3\pi}{4}$ (E) NOTA

2. Compute the dot product of the vectors $\langle 4, 0, -1 \rangle$ and $\langle -3, 9, -3 \rangle$.

(A) -6 (B) 18 (C) 6 (D) -9 (E) NOTA

3. What is the distance between the foci of the hyperbola: $\frac{(x-3)^2}{16} - \frac{(y+1)^2}{12} = 1$.

(A) $\sqrt{7}$ (B) $2\sqrt{7}$ (C) 2 (D) $4\sqrt{7}$ (E) NOTA

4. If $\sin(\theta) = \frac{12}{13}$ find $\cot(\theta)$ if $90^\circ < \theta < 180^\circ$.

(A) $\frac{-5}{12}$ (B) $\frac{-5}{13}$ (C) $\frac{-12}{5}$ (D) $\frac{12}{5}$ (E) NOTA

5. Find the determinant of the matrix

$$M = \begin{bmatrix} 5 & 0 & 1 \\ -2 & 3 & 4 \\ -3 & 2 & 1 \end{bmatrix}$$

(A) -20 (B) -15 (C) -30 (D) -25 (E) NOTA

6. Compute $|2\sqrt{3} + 2\sqrt{6}i|$.

(A) 4 (B) $4\sqrt{2}$ (C) $6\sqrt{2}$ (D) $4\sqrt{3}$ (E) NOTA

7. Simplify $\frac{\tan^2(x) - 1}{\cos(2x)}$ where defined.

(A) $\csc^2(x)$ (B) $\sec(2x)$ (C) $-\sec^2(x)$ (D) $\tan(x)\sec(x)$ (E) NOTA

8. Events A and B are independent. If $P(A \cup B) = 0.80$ and $P(B) = 0.60$, then find $P(A)$.

(A) 0.50 (B) 0.60 (C) 0.55 (D) 0.45 (E) NOTA

9. Solve for x where $x = \sqrt{20 - \sqrt{20 - \sqrt{20 - \dots}}}$

(A) 4 (B) $\sqrt{10}$ (C) $2\sqrt{5}$ (D) 5 (E) NOTA

10. Compute $\lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - x - 2}{x^3 + 5x^2 + 2x - 8}$.

(A) $\frac{-3}{2}$ (B) $\frac{-1}{4}$ (C) $\frac{-1}{2}$ (D) undefined (E) NOTA

11. Which of the following is equal to $\cos(\tan^{-1}(x^2))$?

(A) $\frac{x^2}{\sqrt{x^4 + 1}}$ (B) $\frac{-1}{4}$ (C) $\frac{x^2}{2}$ (D) $\frac{1}{\sqrt{x^4 + 1}}$ (E) NOTA

12. Find the value of $\operatorname{cis}\left(\frac{2\pi}{27}\right)^9 + \operatorname{cis}\left(\frac{7\pi}{15}\right)^5 + \operatorname{cis}\left(\frac{-\pi}{14}\right)^7$.

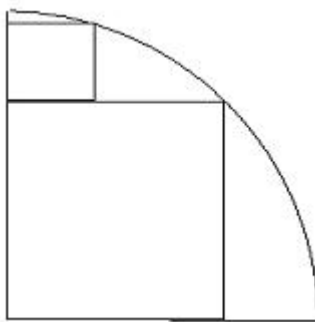
(A) 0 (B) $i\sqrt{3} - i$ (C) $2i$ (D) $i\sqrt{3} + i$ (E) NOTA

13. Identify the shape of the graph of the polar function: $r = 3 \cos \theta - 2 \sin \theta$.
(A) Circle (B) Cardioid (C) Limacon (D) Non-Circular Ellipse (E) NOTA
14. What is the phase shift of the sinusoidal: $y = -2 \sin(4x + 3) - 4$.
(A) $\frac{3}{4}$ (B) $-\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{4}{3}$ (E) NOTA
15. If $f(x) = \frac{6}{(2x-1)^3}$, find $f'(2)$.
(A) $\frac{2}{9}$ (B) $-\frac{8}{9}$ (C) -4 (D) $-\frac{4}{9}$ (E) NOTA
16. If $145_7 + 312_4 = x_6$, find x .
(A) 457 (B) 344 (C) 136 (D) 366 (E) NOTA
17. What is the period of the function $f(x) = \sin^4(x) - \cos^4(x)$?
(A) π (B) $\pi\sqrt{2}$ (C) $\frac{\pi}{2}$ (D) 2π (E) NOTA
18. There are 3 girls and 8 guys in Mrs. McDonald's chemistry class. Mrs. McDonald needs to select 4 students to go to the regional science bowl but she can't decide who she wants to take. How many different teams can go to the science bowl if each team must have at least 1 girl?
(A) 252 (B) 220 (C) 260 (D) 225 (E) NOTA
19. Simplify $\frac{-3+2i}{8-6i} + \frac{2+i}{5-5i}$.
(A) $\frac{13+5i}{50}$ (B) $\frac{30-i}{50}$ (C) $\frac{-1+3i}{25}$ (D) $\frac{-13+14i}{50}$ (E) NOTA
20. Find the sum of all possible values of x for the equation: $|3x-2| + |4x+1| = 20$
(A) 3 (B) $\frac{1}{7}$ (C) $\frac{2}{7}$ (D) $\frac{4}{7}$ (E) NOTA
21. Define a descending number as a number with each digit after the first strictly greater than the digit on the left. For example, 4321 is a descending number but 4532 is not. How many descending numbers are there between 5000 and 7000?
(A) 36 (B) 50 (C) 42 (D) 30 (E) NOTA
22. Find the sum of the infinite series: $\frac{1}{3} + \frac{4}{9} + \frac{7}{27} + \frac{10}{81} + \dots$
(A) $\frac{5}{6}$ (B) $\frac{2}{3}$ (C) $\frac{5}{4}$ (D) $\frac{7}{6}$ (E) NOTA
23. Let S_n indicate the sum of the first n positive integers. Evaluate $\sum_{n=2}^{20} S_n$.
(A) 1539 (B) 1540 (C) 1779 (D) 1780 (E) NOTA

24. Let nondegenerate triangle $\triangle ABC$ have length \overline{AB} equal to 10 and \overline{AC} equal to 12. Let $a, b,$ and c be the sides across angles $\angle A, \angle B$ and $\angle C$ respectively. Compute

$$a^2 \left(\frac{\sin^2(\angle A) + \sin^2(\angle C)}{\sin^2(\angle A)} \right) - 2ac(\cos(\angle B))$$

- (A) 144 (B) 100 (C) 484 (D) 156 (E) NOTA
25. How many solutions within the interval $(0, 2\pi]$ satisfy the equation $\cos(4x) = \cos(9x)$
- (A) 15 (B) 16 (C) 17 (D) 18 (E) NOTA
26. Find the sum of all distinct real values of x within the interval $(0, \pi]$ to the equation: $2 \log_3 \cos 4x - 2 \log_3 \sin 4x = -1$
- (A) $\frac{3\pi}{2}$ (B) 2π (C) $\frac{11\pi}{6}$ (D) $\frac{3\pi}{4}$ (E) NOTA
27. What is the equation of the line that is tangent to the graph $y = x^2 - 3x + 2$ at $x = 4$ (in slope intercept form)?
- (A) $y = 4x + 6$ (B) $y = 6x + 5$ (C) $y = x$ (D) $y = 5x + 6$ (E) NOTA
28. Compute $\lim_{x \rightarrow 0} \frac{1 - \cos(x) - \sin(x)}{x}$
- (A) -2 (B) -1 (C) 0 (D) $-\infty$ (E) NOTA
29. A square is inscribed within a quartercircle of radius 1 as shown below. Another square is drawn that is tangent to the quartercircle and shares a side with the originally inscribed square. What is the side length of the smaller square?



- (A) $\frac{\sqrt{6} - \sqrt{2}}{4}$ (B) $\frac{\sqrt{2}}{4}$ (C) $\frac{\sqrt{2 - \sqrt{2}}}{2}$ (D) $\frac{\sqrt{6} + \sqrt{2}}{4}$ (E) NOTA
30. What is the area of the ellipse defined by the function: $y = 7x^2 - 2\sqrt{3}xy + 5y^2 - 16 = 0$?
- (A) $2\sqrt{3}\pi$ (B) $2\sqrt{2}\pi$ (C) $4\sqrt{2}\pi$ (D) 4π (E) NOTA